Amsterdam University of Applied Sciences | DIGITAL SOCIETY SCHOOL

DATA-DRIVEN TRANSFORMATION

INTRODUCTION TO BAYESIAN THEOREM

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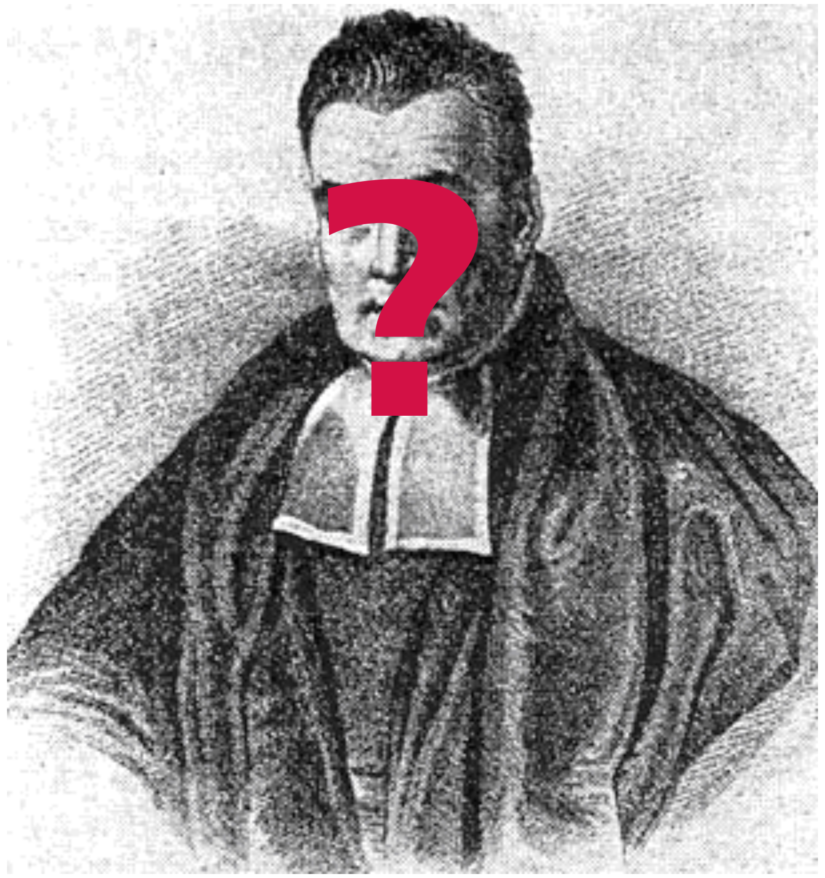
THOMAS BAYES

He postulated the **Bayes theorem**, published 2 years after his death.

"An Essay towards solving a Problem in the Doctrine of Chances" -1763



BAYES THEOREM

Conditional probabilities - P(*this* | *that*), probability *this* is true given *that* is true - have this handy property:

$$p(\mathbf{x})\, p(C_k \mid \mathbf{x}) = p(C_k)\, p(\mathbf{x} \mid C_k)$$

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$$= p(C_k, x)$$

P(*this, that*), probability *this* and *that* are true

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$$p(C_k \mid \mathbf{x}) = \frac{p(C_k)\, p(\mathbf{x} \mid C_k)}{p(\mathbf{x})}$$

BAYES THEOREM

Conditional probabilities - P(_this_ | _that_), probability _this_ is true given _that_ is true - have this handy property:

$$\underbrace{p(C_k \mid \mathbf{x})}_{\text{Posterior}} = \frac{\overbrace{p(C_k)}^{\text{Prior}}\,\overbrace{p(\mathbf{x} \mid C_k)}^{\text{Likelihood}}}{\underbrace{\boxed{p(\mathbf{x})}}_{\text{Normalizing Constant}}}$$

EXAMPLE

- You get tested for malaria once back from India.
- **You got malaria**, says your test :(
- When classifying tourists (malaria or not) the **test has only 2% error** :(
- 1% of tourists back from India have malaria.

Oh no… Your surely have malaria, like 98% chance!

EXAMPLE

$$\underbrace{p(C_k \mid \mathbf{x})}_{\text{Posterior}} = \frac{\overbrace{p(C_k)}^{\text{Prior}}\;\overbrace{p(\mathbf{x} \mid C_k)}^{\text{Likelihood}}}{p(\mathbf{x})}$$

Posterior

Prior

Likelihood

having malaria   test is positive

EXAMPLE

Do you have malaria when the test is positive?

$$\underbrace{p(C_k \mid \mathbf{x})}_{\text{Posterior}} = \frac{\underbrace{p(C_k)}_{\text{Prior}}\ \underbrace{p(\mathbf{x} \mid C_k)}_{\text{Likelihood}}}{p(\mathbf{x})}$$

having malaria   test is positive

EXAMPLE

Do you have malaria when the test is positive?

1% of tourists got malaria

Posterior / Prior / Likelihood

$$p(C_k \mid \mathbf{x}) = \frac{p(C_k)\, p(\mathbf{x} \mid C_k)}{p(\mathbf{x})}$$

having malaria — test is positive

EXAMPLE

Do you have malaria when the test is positive?

1% of tourists got malaria

98% of the tests are accurate

Posterior

Prior

Likelihood

$$p(C_k \mid \mathbf{x}) = \frac{p(C_k)\, p(\mathbf{x} \mid C_k)}{p(\mathbf{x})}$$

having malaria

test is positive

EXAMPLE

p(tested positive, malaria)

0.01 x 0.98 = 0.0098

0.98% chance of having malaria AND being tested positive (~1%)

1% of tourists got malaria

98% of the tests are accurate

Do you have malaria when the test is positive?

Posterior

Prior

Likelihood

$$p(C_k \mid \mathbf{x}) = \frac{p(C_k)\, p(\mathbf{x} \mid C_k)}{p(\mathbf{x})}$$

having malaria

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having malaria test is positive

p(tested positive) =
p(tested positive, malaria)
+ p(tested positive, not malaria)

EXAMPLE

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having malaria   test is positive

p(tested positive) =
p(tested positive, malaria)
+ p(tested positive, not malaria)

0.98 x 0.01 + 0.02 x 0.99 = 0.0098 + 0.0198
2.96% chance of being tested positive (~3%)

EXAMPLE

p(tested positive, malaria)

0.01 x 0.98 = 0.0098
0.98% chance of having malaria AND being tested positive (~1%)

Do you have malaria when the test is positive?

1% of tourists got malaria

98% of positive tests are accurate

Posterior | Prior | Likelihood

$$p(C_k \mid \mathbf{x}) = \frac{p(C_k)\, p(\mathbf{x} \mid C_k)}{p(\mathbf{x})}$$

having malaria   test is positive

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Prior

Likelihood

$$\frac{p(C_k)\, p(\mathbf{x} \mid C_k)}{p(\mathbf{x})}$$

p(tested positive) =
p(tested positive, malaria)
+ p(tested positive, not malaria)

Do you have malaria when the test is positive?

You “only” have ~ 1 / 3 chance of having malaria

0.98 x 0.01 + 0.02 x 0.99 = 0.0098 + 0.0198
2.96% chance of being tested positive (~3%)

EXAMPLE

- You try to detect fraudsters through their bank records.
- **We’ve got a great classifier**, says your vendor…
- When classifying **fraudsters** the AI has **only 1% errors** :D
- When classifying **innocents**, the AI has **only 2% errors** :)
- From prior inspections, you expect that 0.5% of your clients are fraudsters.

Oh great software!
When I detect a fraudster, I’ve like 2% error to expect…

EXAMPLE

p(tested fraudster, fraudster)

0.005 x 0.99 = 0.00495
0.495% chance of being a fraudster AND being detected (~0.5%)

Chance of being fraudster (Ck) when detected as fraudster?

0.5% of clients are fraudster

99% of the tests are accurate

$$\underbrace{p(C_k \mid \mathbf{x})}_{\text{Posterior}} = \frac{\underbrace{p(C_k)}_{\text{Prior}}\ \underbrace{p(\mathbf{x} \mid C_k)}_{\text{Likelihood}}}{p(\mathbf{x})}$$

p(fraudster | detected fraudster)

p(detected fraudster) =
p(detected fraudster, fraudster)
+ p(detected fraudster, innocent)

0.005 x 0.99 + 0.995 x 0.02 = 0.00495 + 0.0199
2.96% chance of being detected as fraudster (~2.5%)

Detected as fraudster?
only ~20% chance you are one…
(actually 18%)

You have ~80% chance of being innocent!
(actually 82%)

p(tested fraudster, fraudster)

0.005 x 0.99 = 0.00495
0.495% chance of being a fraudster AND being detected (~0.5%)

when detected as fraudster?

0.5% of clients are fraudster

99% of the tests are accurate

Posterior, Prior, Likelihood

$$p(C_k \mid \mathbf{x}) = \frac{p(C_k)\, p(\mathbf{x} \mid C_k)}{p(\mathbf{x})}$$

p(fraudster | detected fraudster)

p(detected fraudster) =
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QUESTION & DISCUSSION