Amsterdam University of Applied Sciences | DIGITAL SOCIETY SCHOOL

FITTING REGRESSION

EMMA BEAUXIS-AUSSALET

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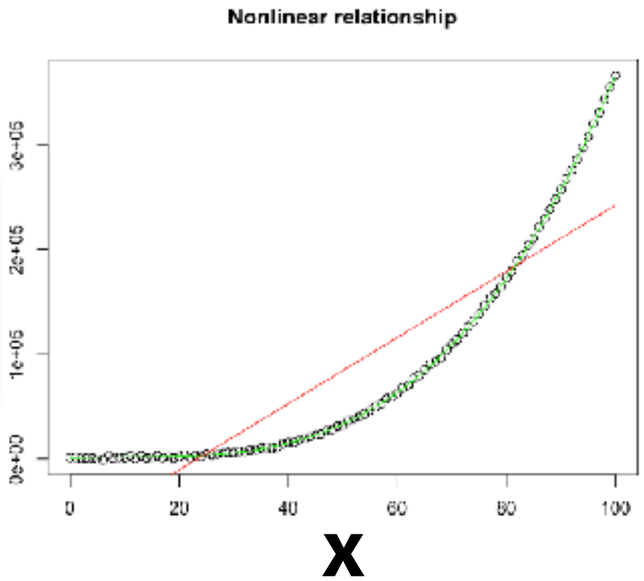
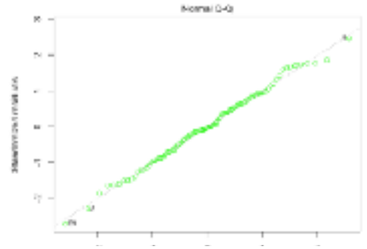
UNDER FITTING

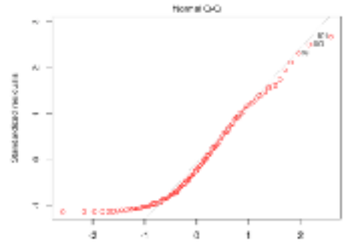
Regressions underfit the data when **essential patterns are not modelled**.
The **residuals are not random** w.r.t. the predictor or predicted variables
(e.g., not normally distributed with mean 0).

$\hat{y} = a_1\mathbf{x} + a_2\mathbf{x^2} + a_3\mathbf{x^3} + b$

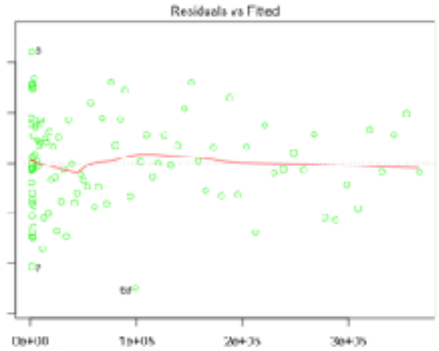
$\hat{y} = a\mathbf{x} + b$





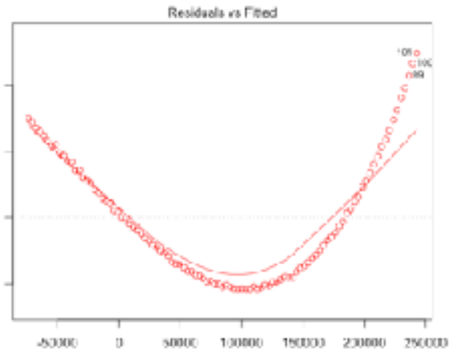


$y - \hat{y}$



$\hat{y} = a_1\mathbf{x} + a_2\mathbf{x^2} + a_3\mathbf{x^3} + b$

$y - \hat{y}$



$\hat{y} = a\mathbf{x} + b$

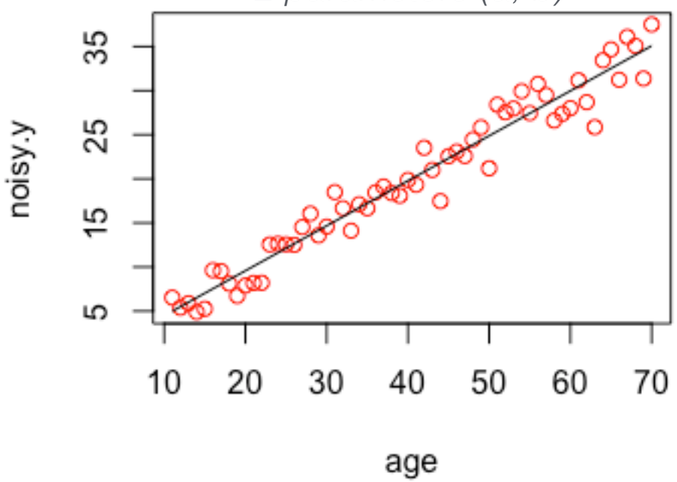
OVER FITTING

Regressions overfit the data when **sample-specific patterns are modelled**.
The **residuals may be low and random** (normally distributed with mean 0).

*More and more **complex regressions**, with more parameters to fit, and **higher risk of overfitting**…*

Simple linear (Degree 1)

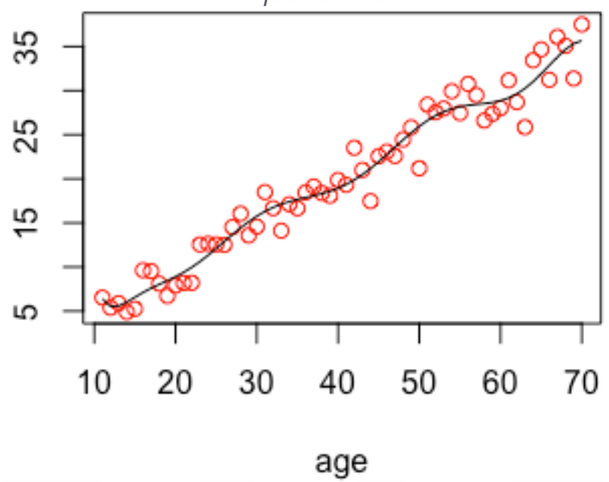
2 parameters (a, b)



$$\hat{y} = a\mathbf{x} + b$$

Degree 10

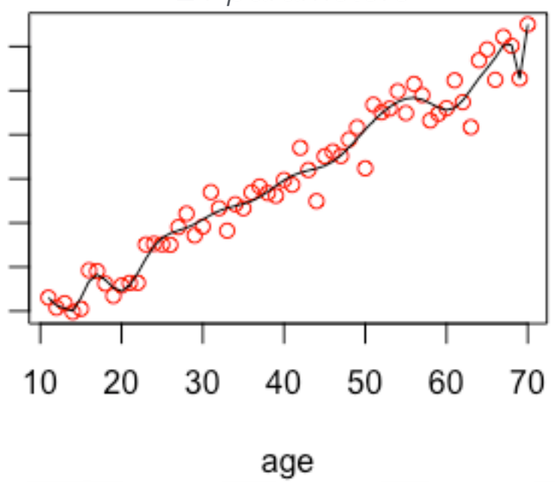
11 parameters



$$\hat{y} = a_1\mathbf{x} + a_2\mathbf{x^2} + \ldots + a_{10}\mathbf{x^{10}} + b$$

Degree 20

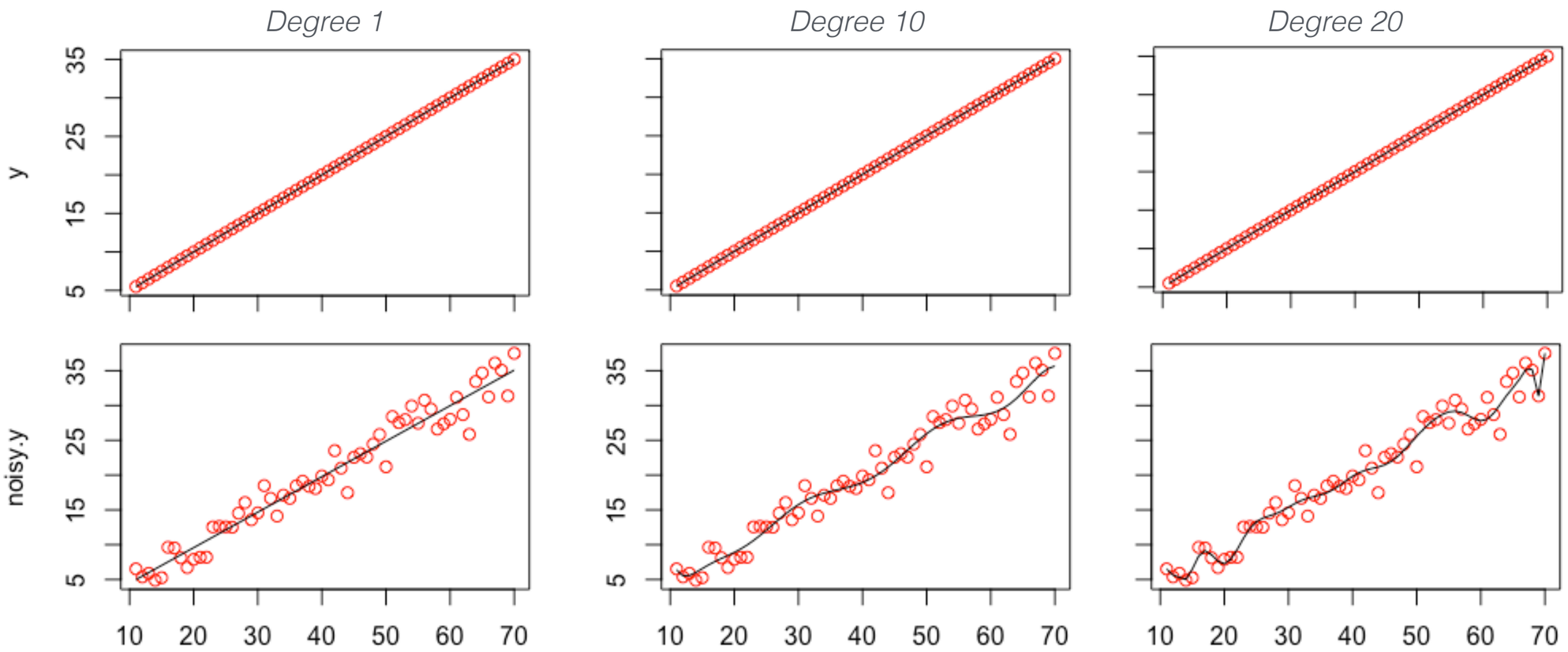
21 parameters



$$\hat{y} = a_1\mathbf{x} + a_2\mathbf{x^2} + \ldots + a_{20}\mathbf{x^{20}} + b$$

NOISE

The risk of overfitting arise from **naturally occurring random variations** (i.e., noise).



PREDICTION VERSUS DESCRIPTION

Under-fitted models have **poor predictive & descriptive properties**.

Over-fitted models have **poor predictive properties** but **better descriptive properties**.

UNDER FITTING

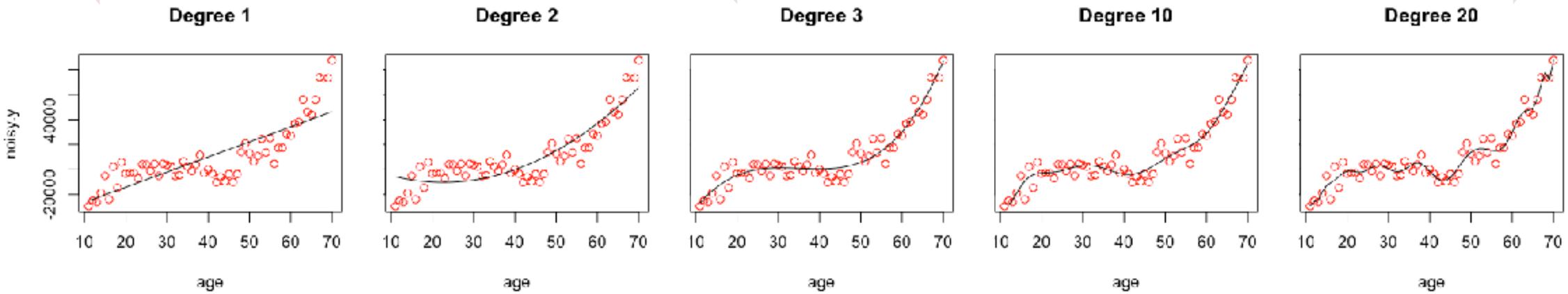
Less predictive, less descriptive…

GOOD FIT

More predictive…

OVER FITTING

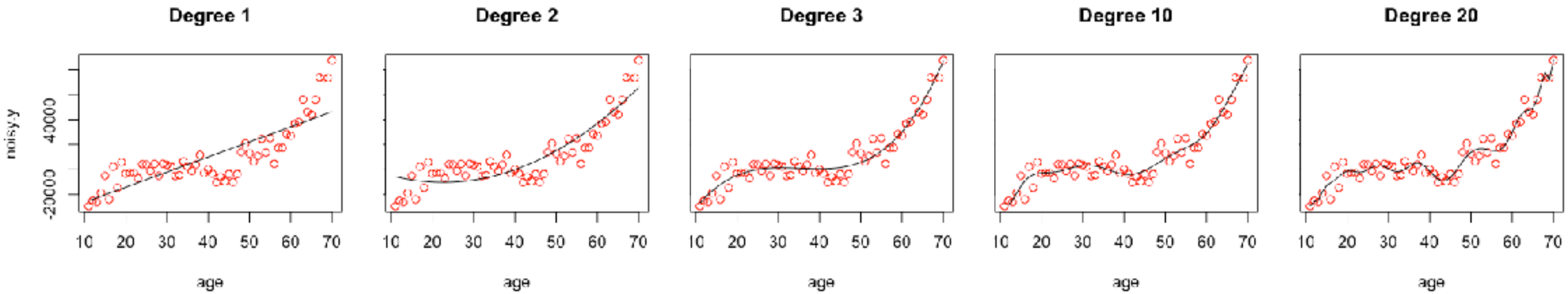
…more descriptive



R CODE

```r
###### Simple linear regression
model <- lm(y ~ age)
plot(model)
```

```r
###### Polynomial degreee 20
model <- lm(y ~ poly(age,20))
```



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REGRESSION WITH ORDINAL DATA

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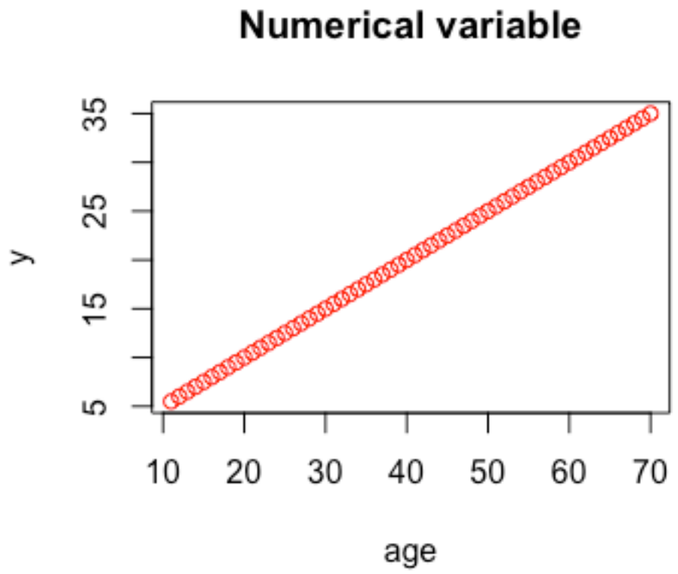
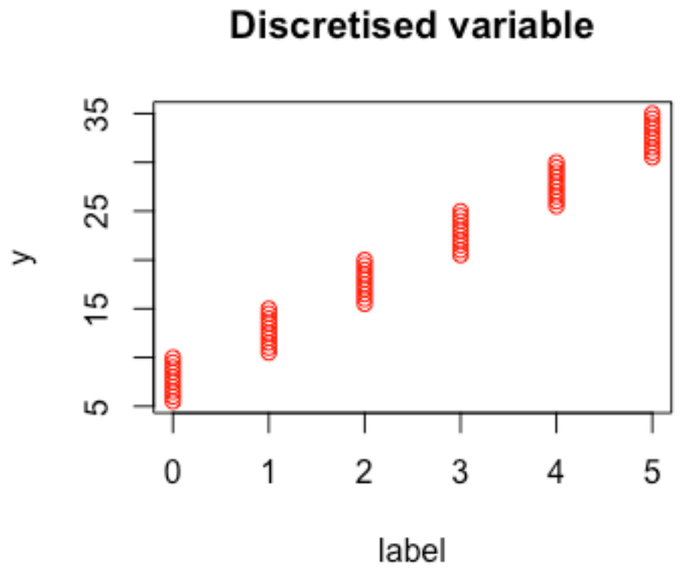
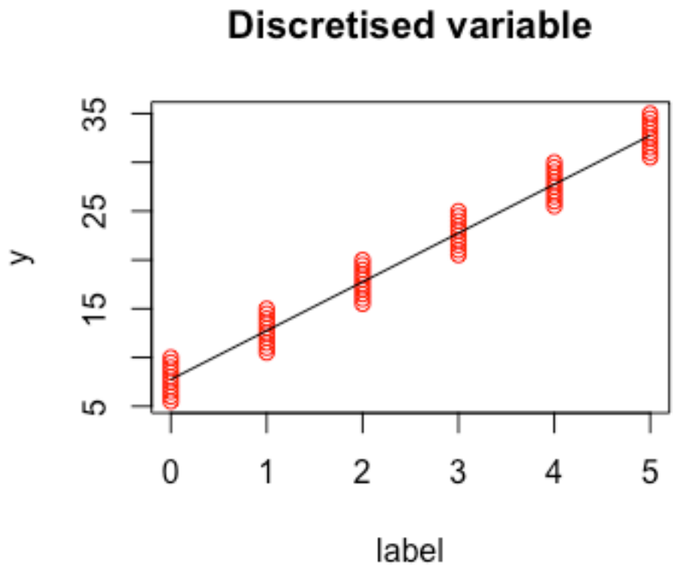
ORDINAL DATA

Regressions can be fitted on ordered categorical variables by using **integer instead of categories**.





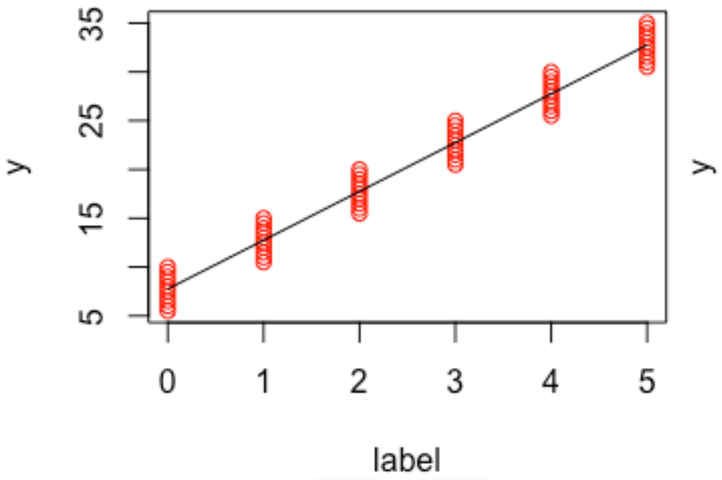



```
###### Linear regression
model <- lm(y ~ label)
```

ORDINAL DATA

Models fitted on discretised data can be reused with numerical data, assuming the patterns are consistent.

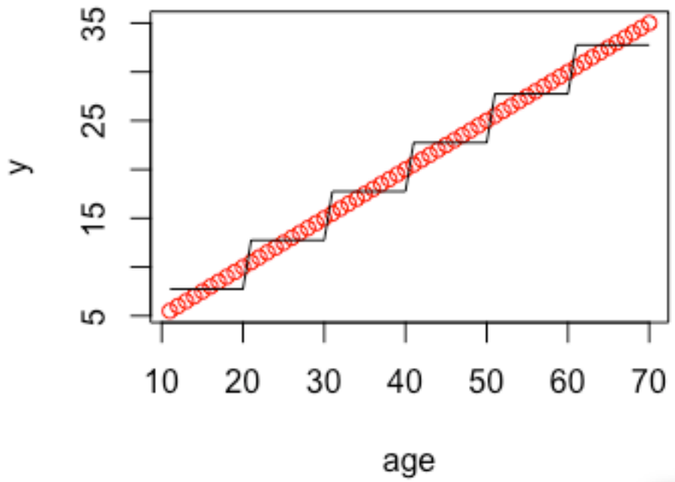
Discretised variable



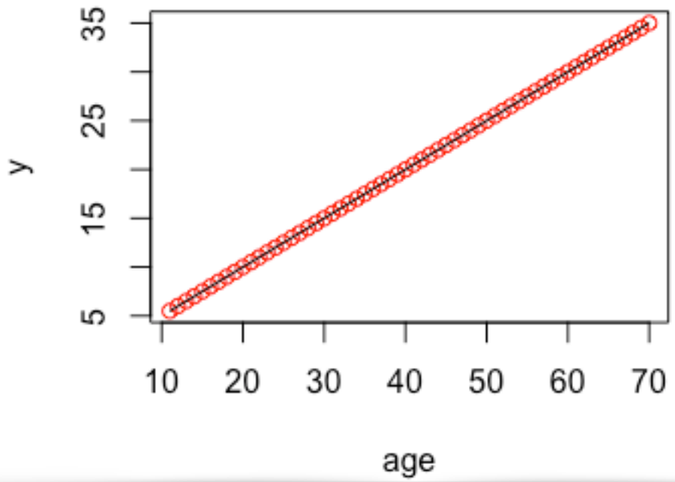

```
Call:
lm(formula = y ~ label)

Coefficients:
(Intercept)        label
       7.75         5.00
```

Numerical variable



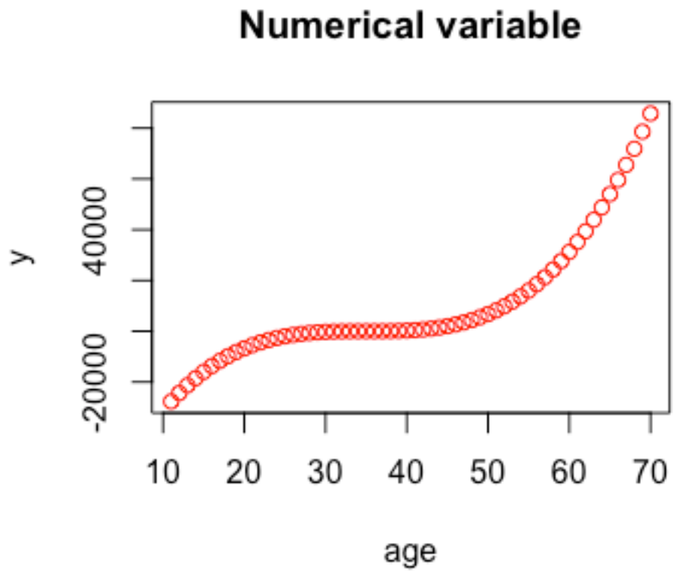
Numerical variable

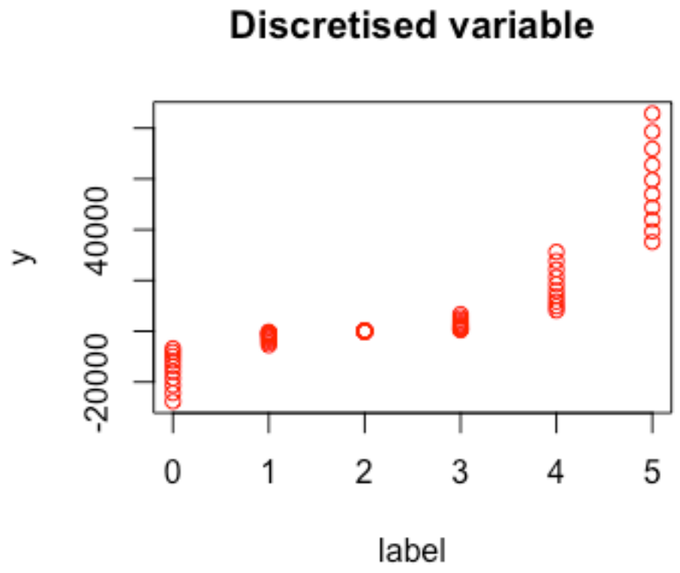


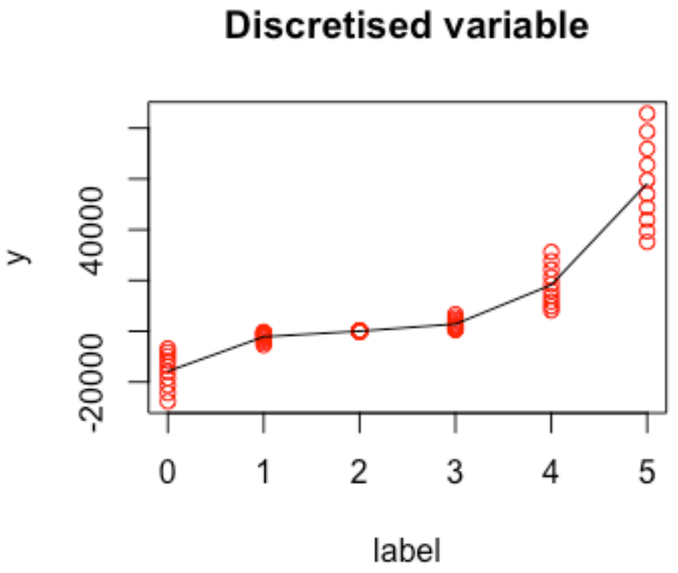

```
plot(age, y, type='p', col='red', main='Numerical variable')
y_hat = 5*(age/10-1.55) + 7.75
lines(age, y_hat)
```

ORDINAL DATA

Regressions can be fitted on ordered categorical variables (e.g., discretised data) by using **integer instead of categories**.



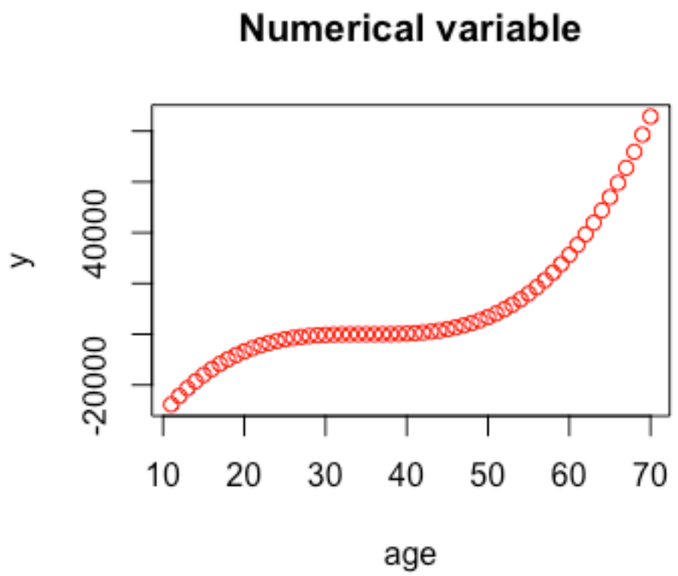


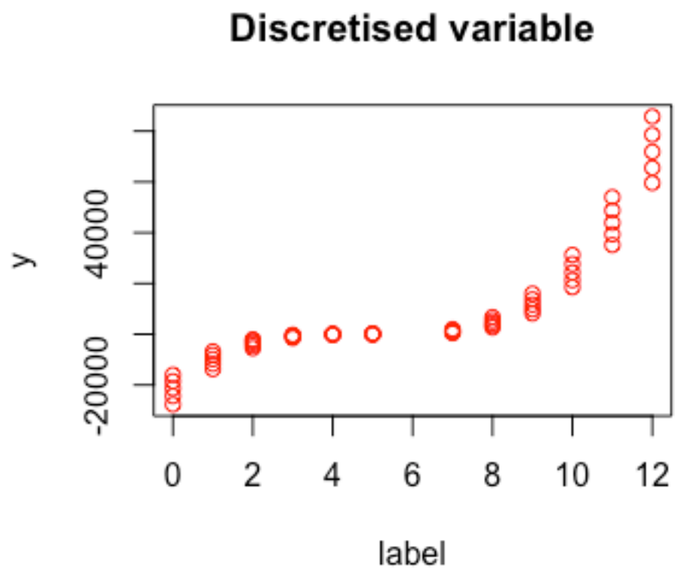


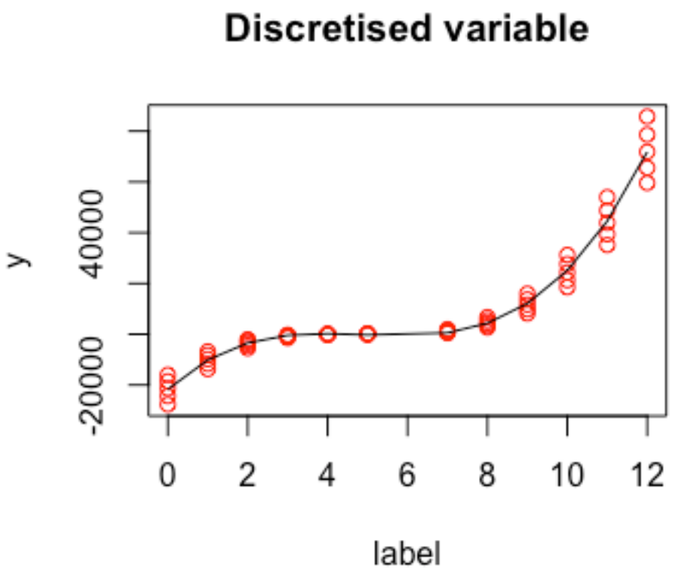

```
###### Polynomial regression - Degree 3
model <- lm(y ~ poly(label,3))
```

ORDINAL DATA

The granularity of discretisation (i.e., bin width) impacts the uncertainty.



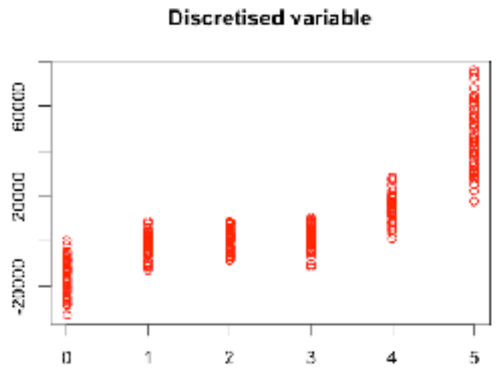


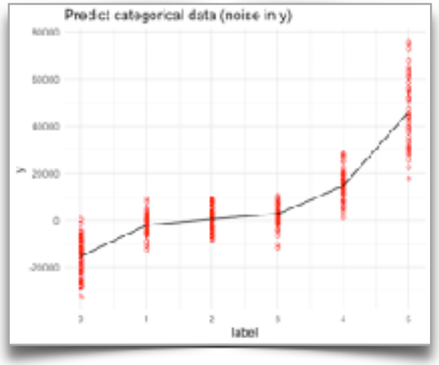


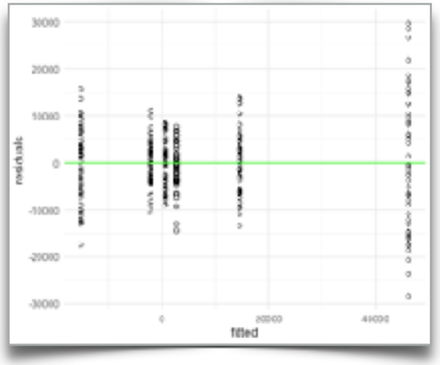

```
###### Polynomial regression - Degree 3
model <- lm(y ~ poly(label,3))
```

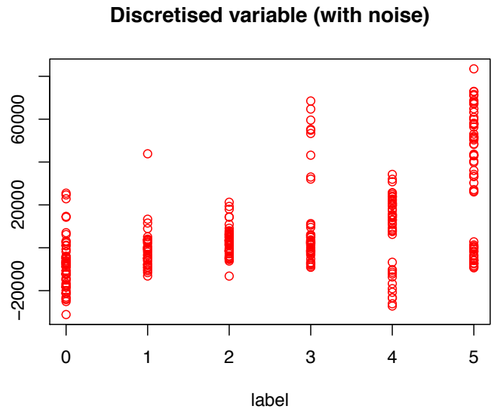
ORDINAL DATA

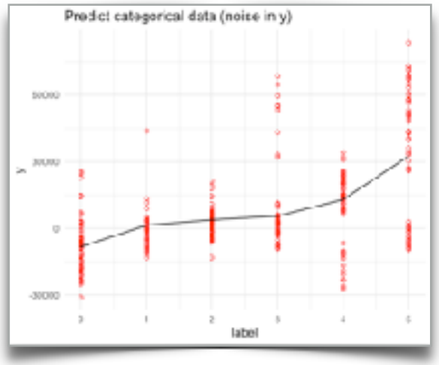
Residuals are also discretised, and subject to noise and heteroscedasticity.

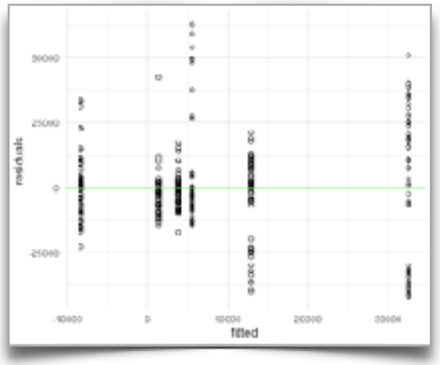












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REGRESSION WITH CATEGORICAL DATA

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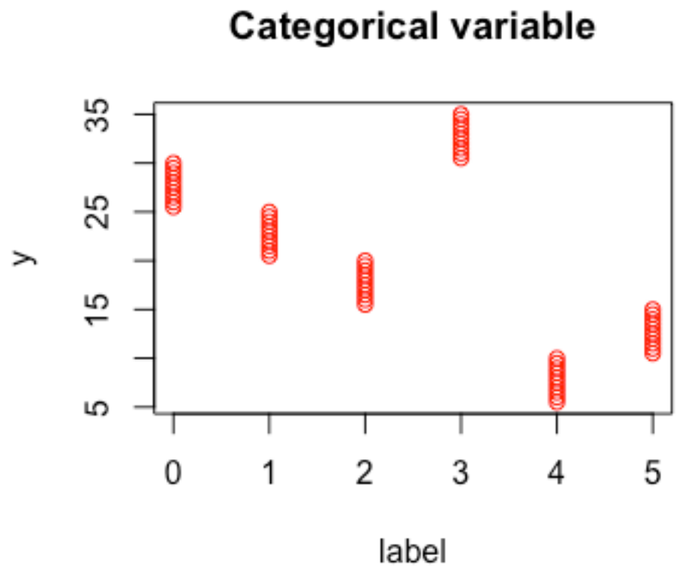
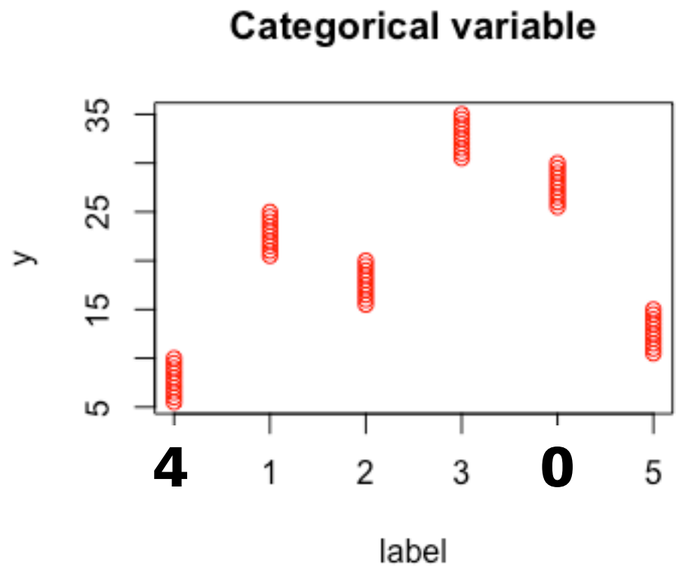
UNORDERED CATEGORIES

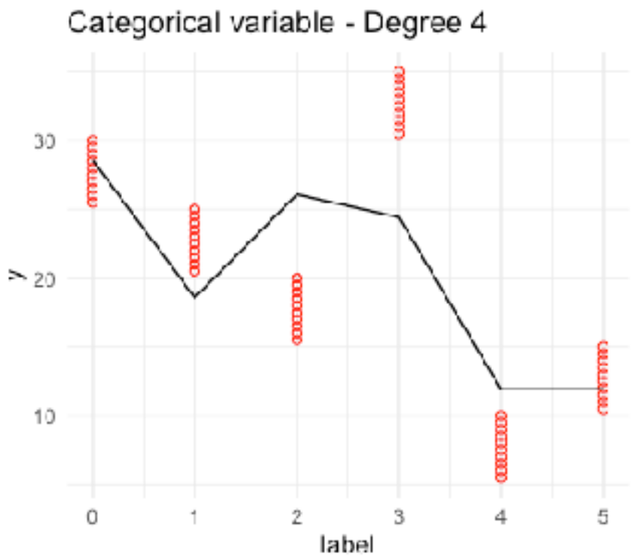
When transforming multiple categories into numerical index, the **patterns are arbitrary**.

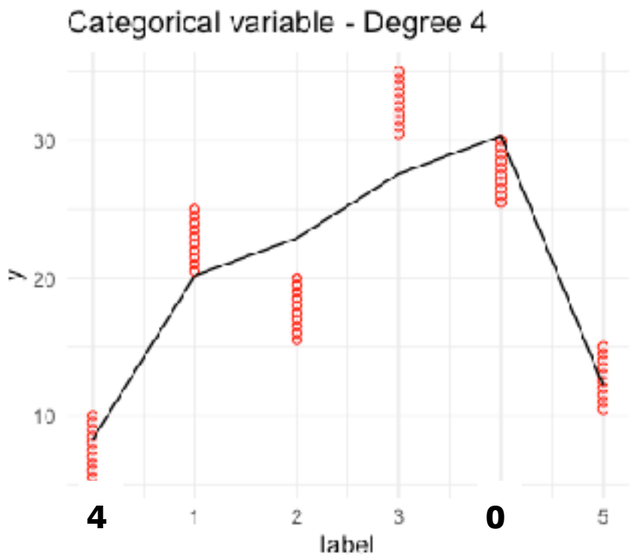




UNORDERED CATEGORIES

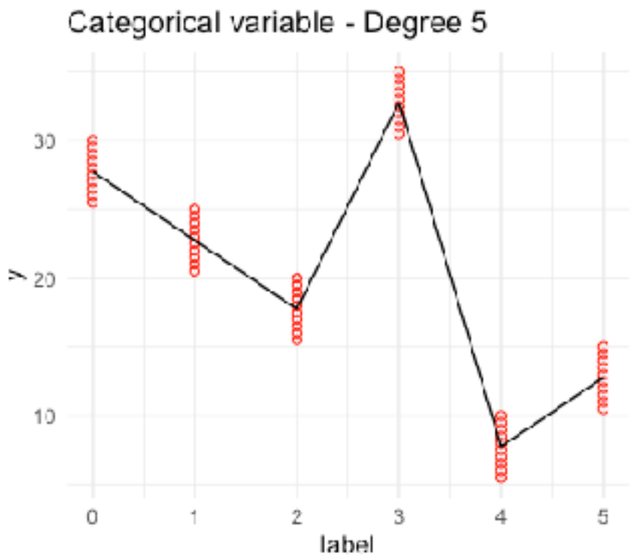
When transforming multiple categories into numerical index, the **patterns are arbitrary**. **Models are biased**.

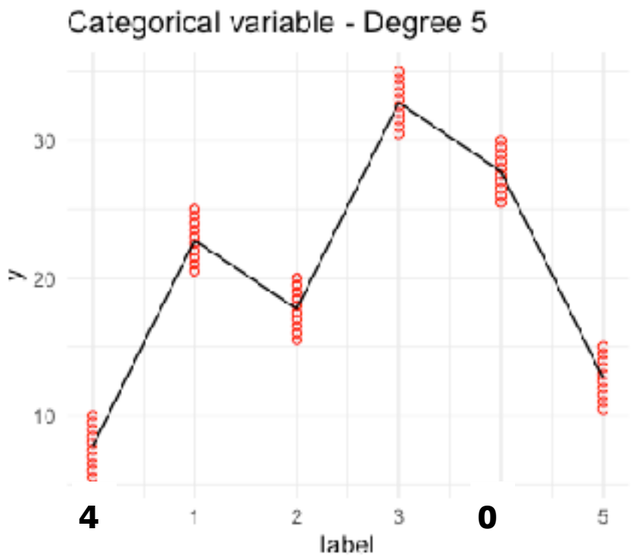




UNORDERED CATEGORIES

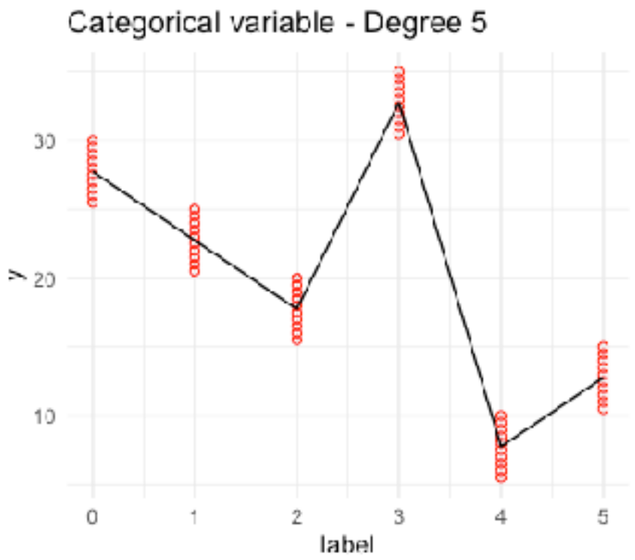
When transforming multiple categories into numerical index, the **patterns are arbitrary**. **Models are biased**… unless completely overfitting!

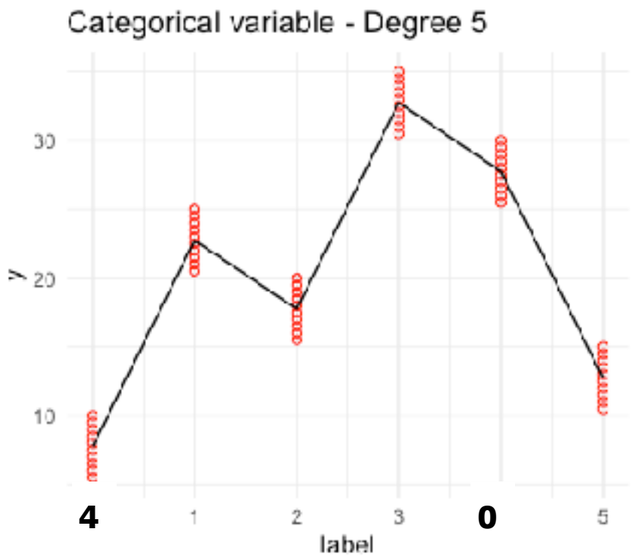




UNORDERED CATEGORIES

When transforming multiple categories into numerical index, the **patterns are arbitrary**.
Models are biased… unless completely overfitting!
And they remain **uninterpretable**, and an issue for multivariate regressions.





DUMMY CODING

Categorical data must be **transformed into "*wide*" data** with a column per category. For each data point, the **category is indicated with 1 or 0** (in category, or not).

```
> data_narrow[1:15,]
   label    y
1      A  5.5
2      A  6.0
3      A  6.5
4      A  7.0
5      A  7.5
6      A  8.0
7      A  8.5
8      A  9.0
9      A  9.5
10     A 10.0
11     F 10.5
12     F 11.0
13     F 11.5
14     F 12.0
15     F 12.5
```

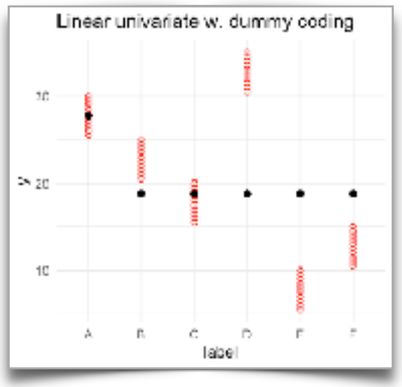
```
> data_narrow_prep[1:15,]
   id label    y dummy
1   1     A  5.5     1
2   2     A  6.0     1
3   3     A  6.5     1
4   4     A  7.0     1
5   5     A  7.5     1
6   6     A  8.0     1
7   7     A  8.5     1
8   8     A  9.0     1
9   9     A  9.5     1
10 10     A 10.0     1
11 11     F 10.5     1
12 12     F 11.0     1
13 13     F 11.5     1
14 14     F 12.0     1
15 15     F 12.5     1
```

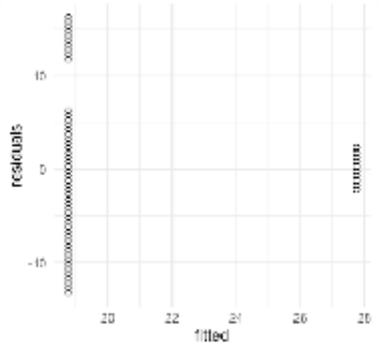
```
> data_wide <- data_narrow_prep %>% spread(label, dummy, fill=0)
> data_wide[1:15,]
   id    y A B C D E F
1   1  5.5 1 0 0 0 0 0
2   2  6.0 1 0 0 0 0 0
3   3  6.5 1 0 0 0 0 0
4   4  7.0 1 0 0 0 0 0
5   5  7.5 1 0 0 0 0 0
6   6  8.0 1 0 0 0 0 0
7   7  8.5 1 0 0 0 0 0
8   8  9.0 1 0 0 0 0 0
9   9  9.5 1 0 0 0 0 0
10 10 10.0 1 0 0 0 0 0
11 11 10.5 0 0 0 0 0 1
12 12 11.0 0 0 0 0 0 1
13 13 11.5 0 0 0 0 0 1
14 14 12.0 0 0 0 0 0 1
15 15 12.5 0 0 0 0 0 1
```

DUMMY CODING

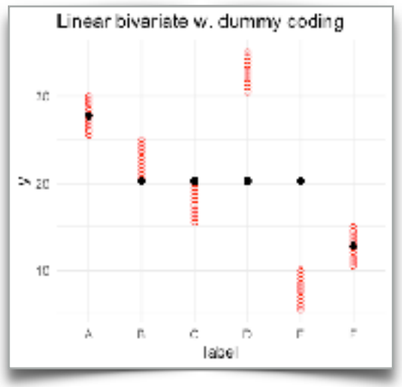
Multivariate linear regressions can accurately fit dummy coded variables.

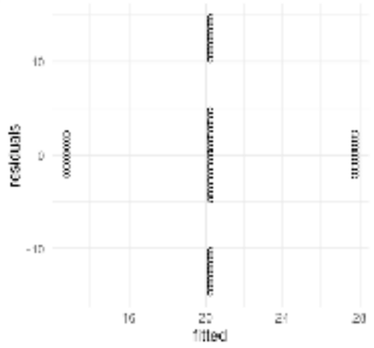
```
########## Univariate regression
model <- lm(y ~ A, data=data_wide)
```



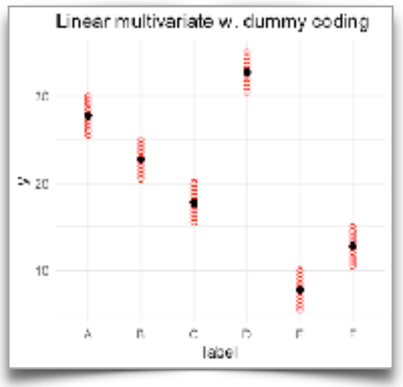


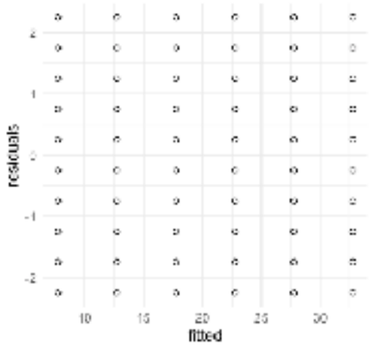

```
########## Bivariate regression
model <- lm(y ~ A+F, data=data_wide)
```






```
########## Multivariate regression
model <- lm(y ~ A+B+C+D+E+F, data_wide)
```

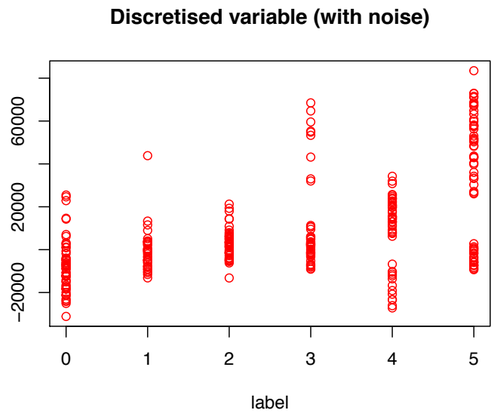


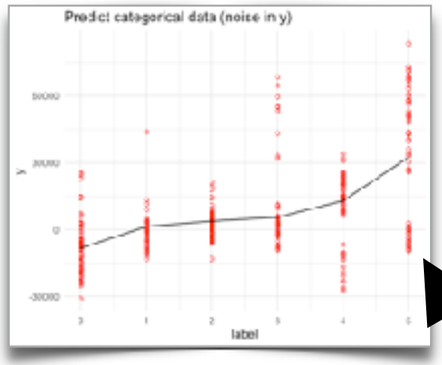


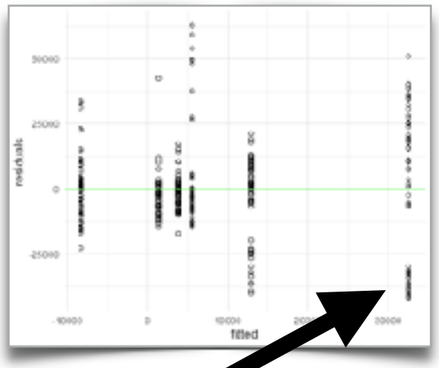
	id	y	A	B	C	D	E	F
1	1	5.5	1	0	0	0	0	0
2	2	6.0	1	0	0	0	0	0
3	3	6.5	1	0	0	0	0	0
4	4	7.0	1	0	0	0	0	0
5	5	7.5	1	0	0	0	0	0
6	6	8.0	1	0	0	0	0	0
7	7	8.5	1	0	0	0	0	0
8	8	9.0	1	0	0	0	0	0
9	9	9.5	1	0	0	0	0	0
10	10	10.0	1	0	0	0	0	0
11	11	10.5	0	0	0	0	0	1
12	12	11.0	0	0	0	0	0	1
13	13	11.5	0	0	0	0	0	1
14	14	12.0	0	0	0	0	0	1
15	15	12.5	0	0	0	0	0	1

ORDINAL DATA

Dummy coding is also recommended for ordinal data and multivariate regression.







The variable interactions creating these patterns would be better modelled with dummy coding.

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QUESTIONS?

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