

DATA-DRIVEN TRANSFORMATION

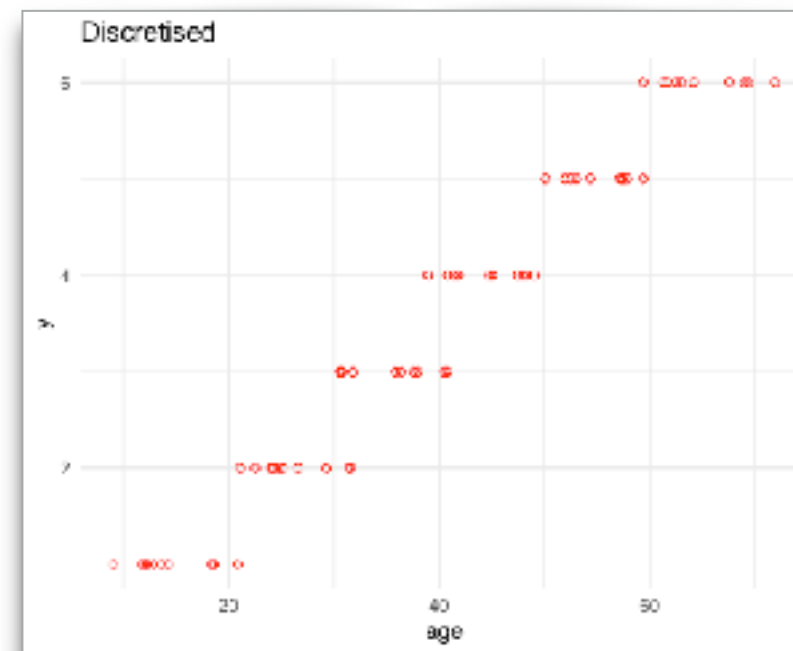
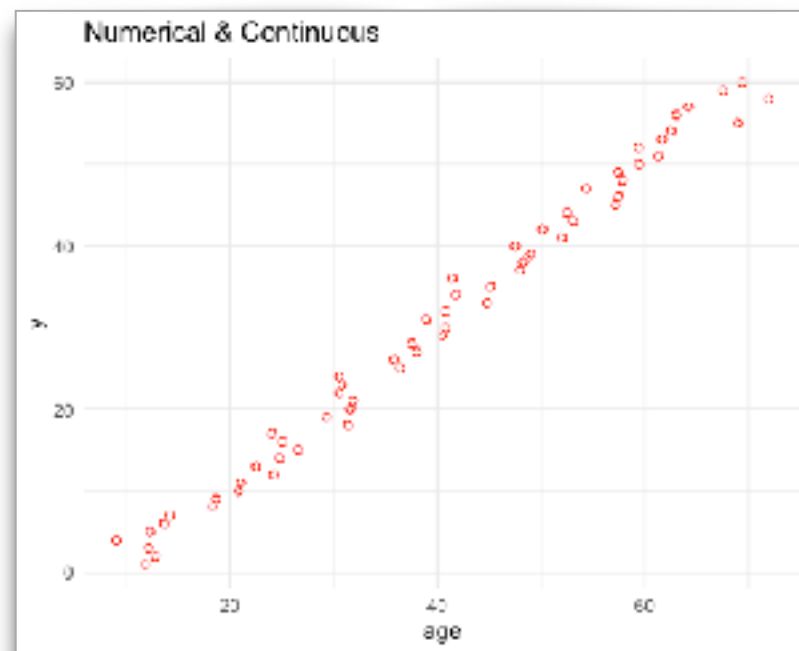
PREDICTING ORDINAL DATA

EMMA BEAUXIS-AUSSALET

e.m.a.l.beauxis@hva.nl

ORDINAL DATA

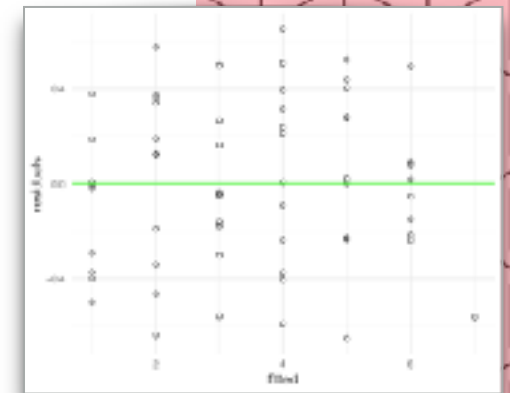
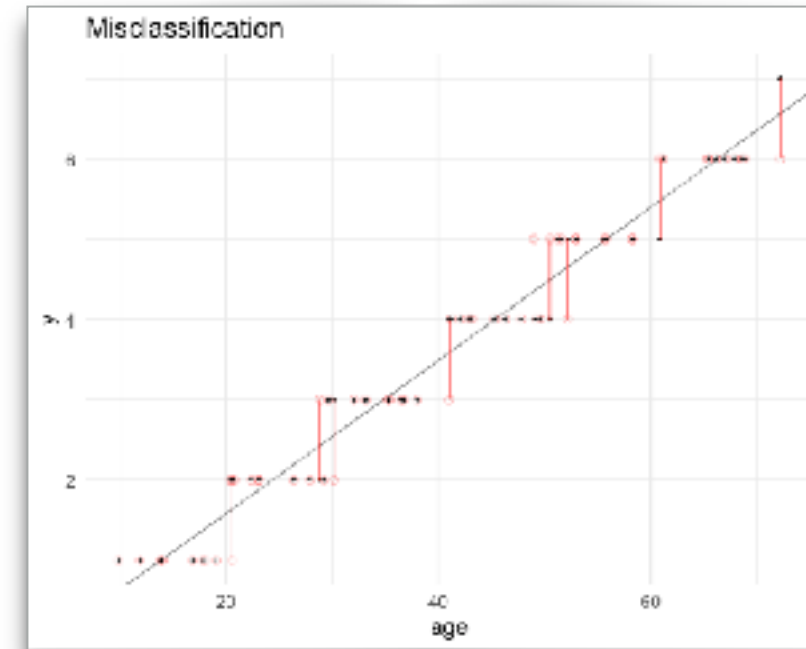
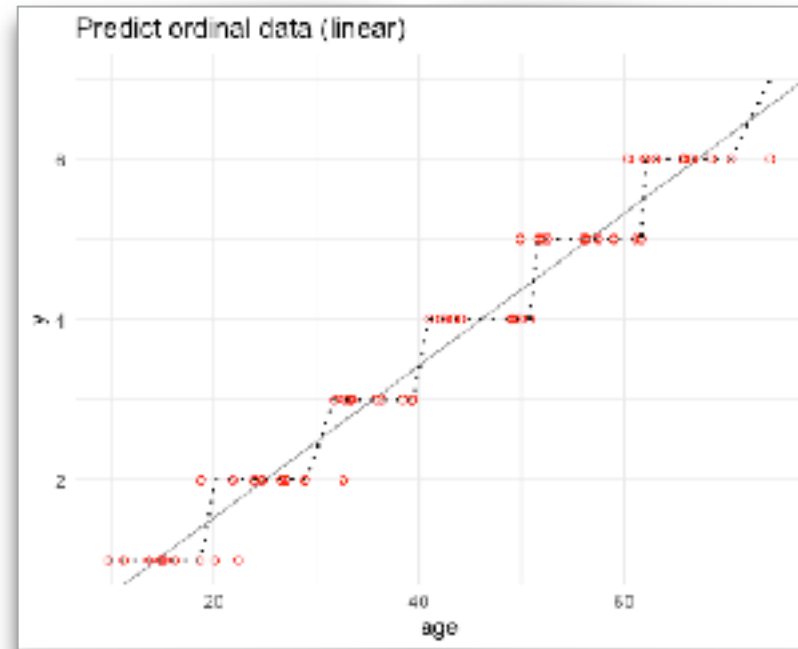
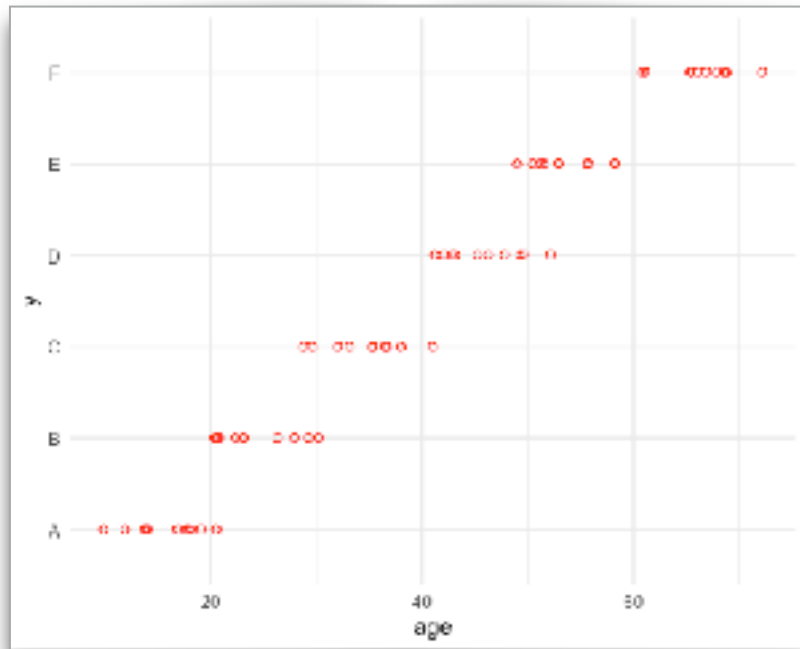
Ordinal data have continuous relationship between their 'categories'.
For instance, numerical data are often discretised (e.g., ratings).



PREDICT ORDINAL DATA

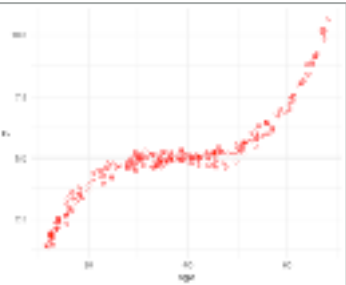
Linear regressions can be fitted on ordinal data.

```
##### Linear regression  
model <- lm(y ~ age)
```

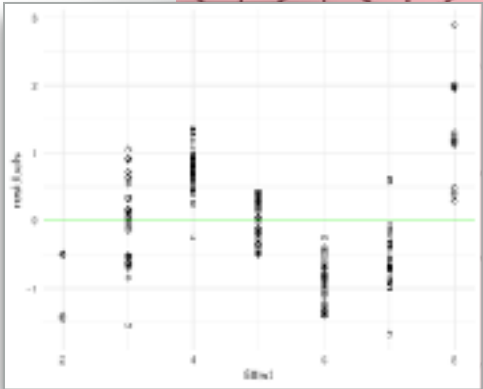
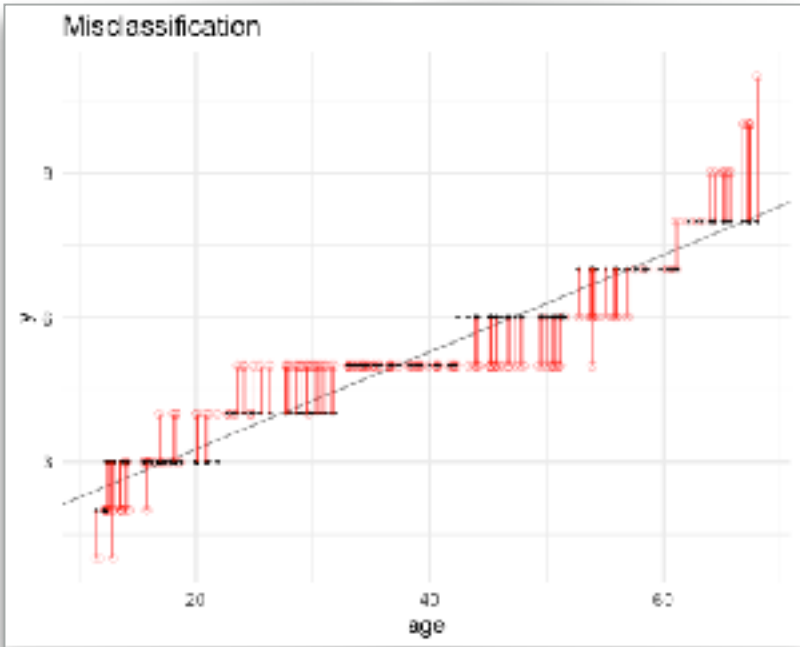
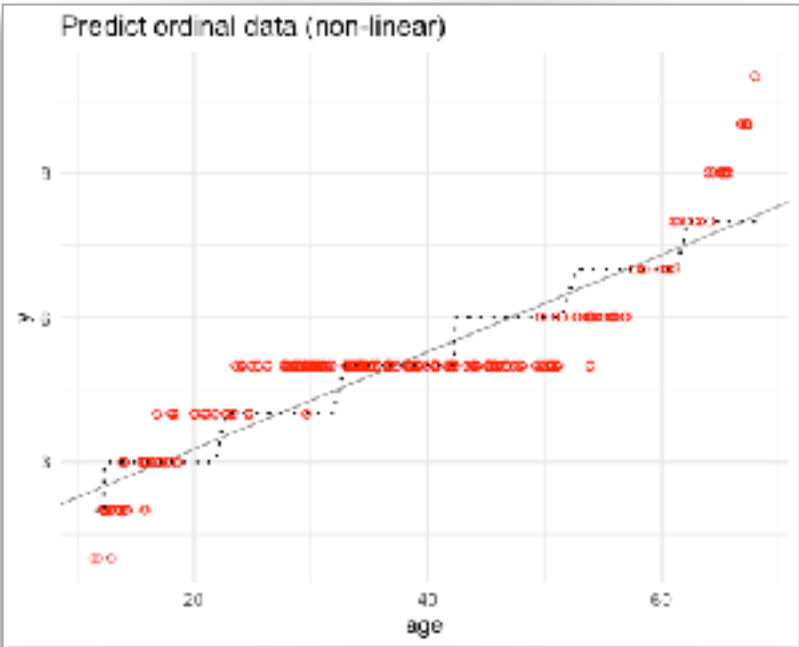
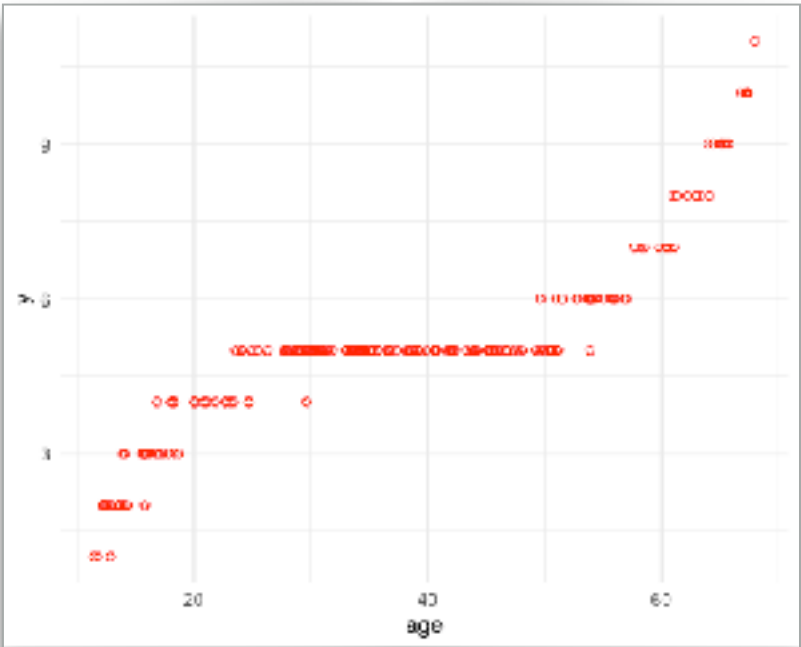


PREDICT ORDINAL DATA

Linear regressions can be fitted on ordinal data.

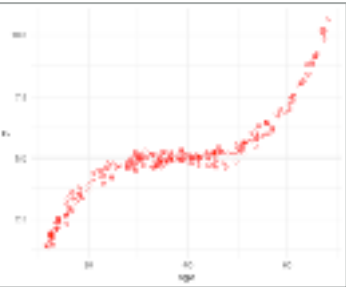


```
##### Linear regression
model <- lm(y ~ age)
```

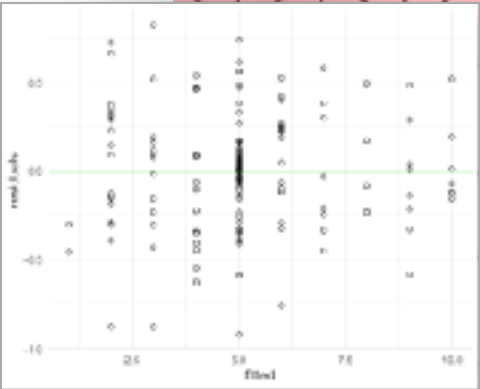
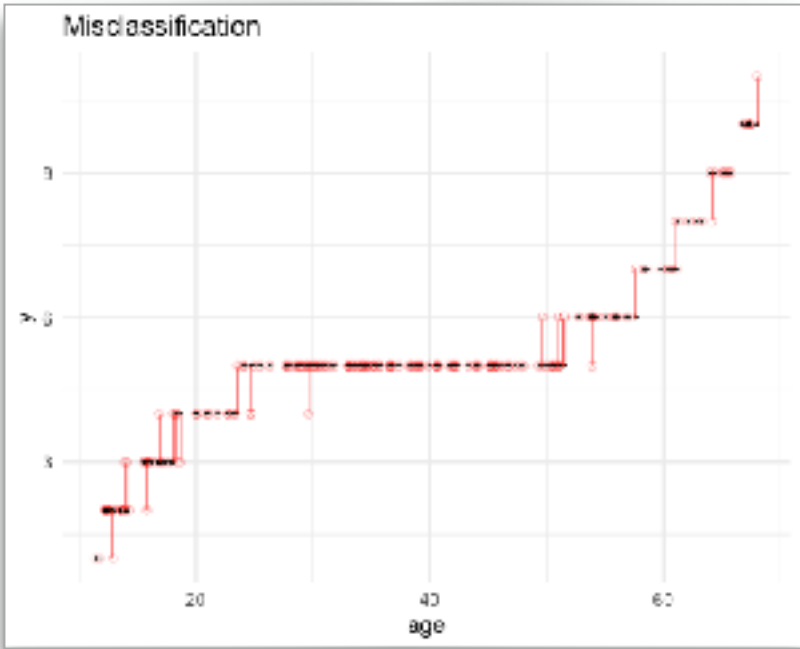
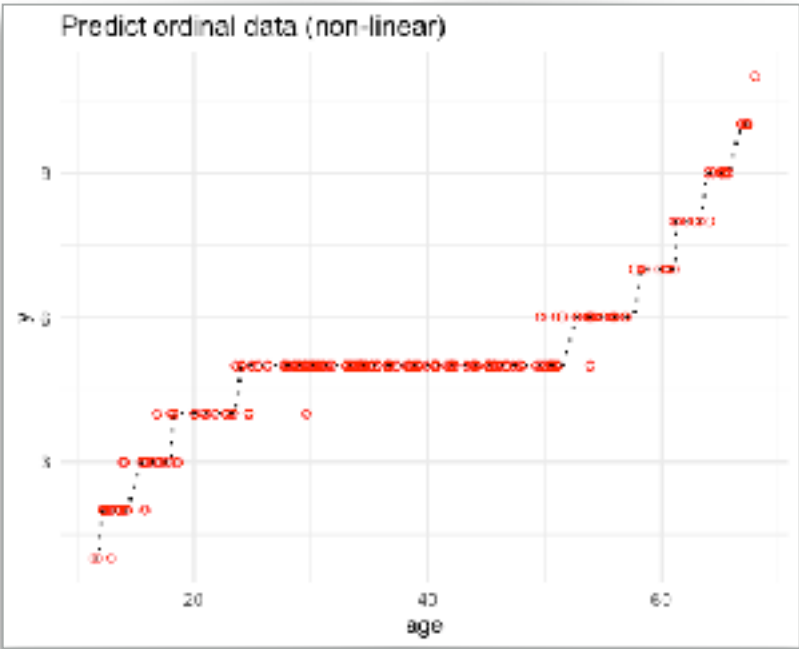
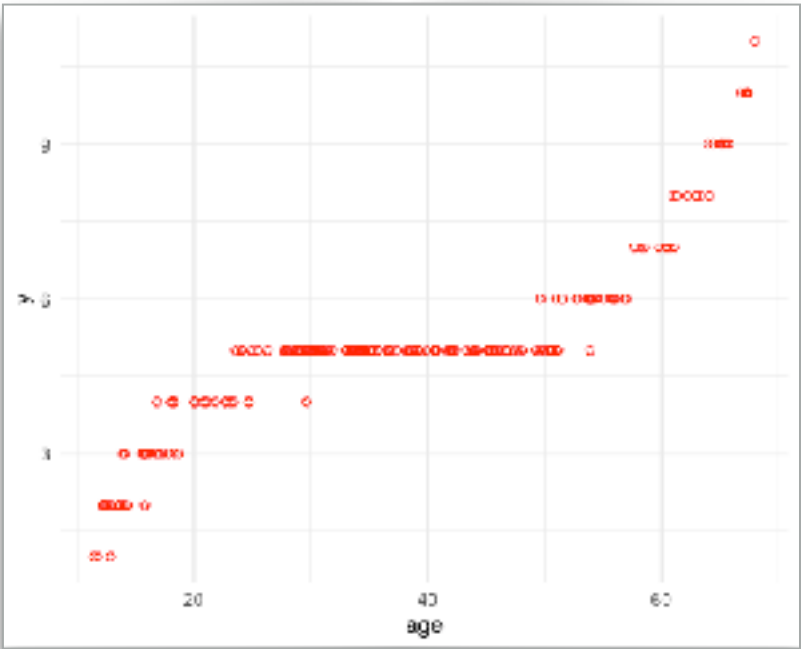


PREDICT ORDINAL DATA

Linear regressions can be fitted on ordinal data.



```
##### Polynomial regression
model <- lm(y ~ poly(age,3))
```



DATA-DRIVEN TRANSFORMATION

PREDICTING CATEGORICAL DATA

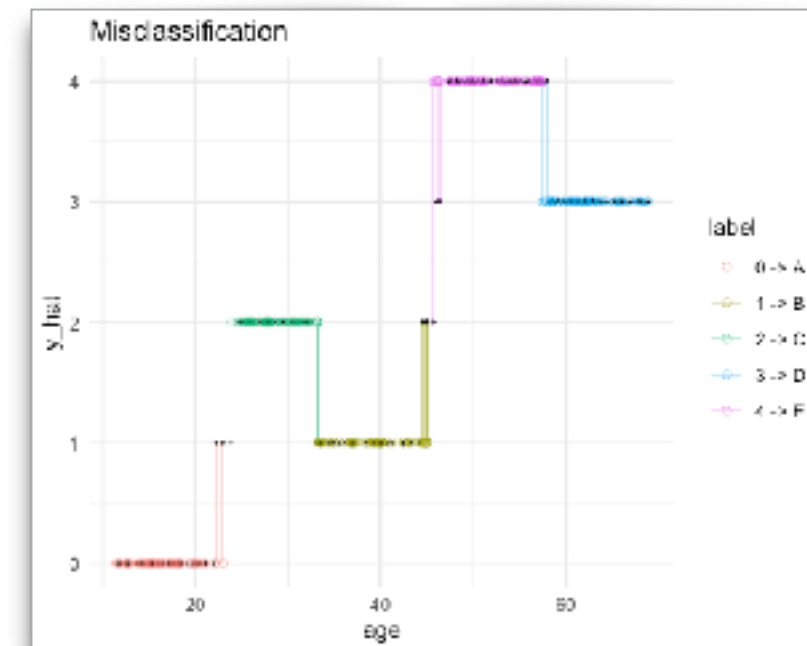
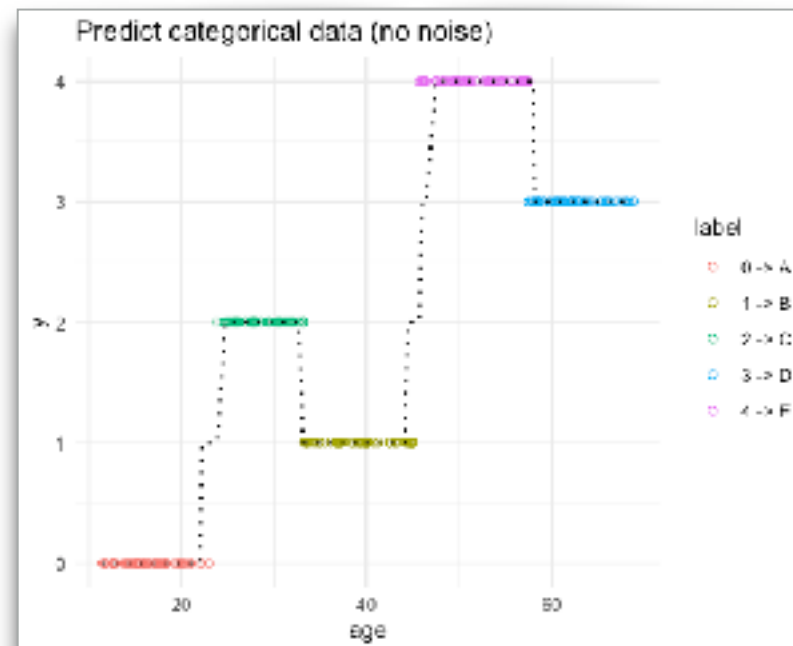
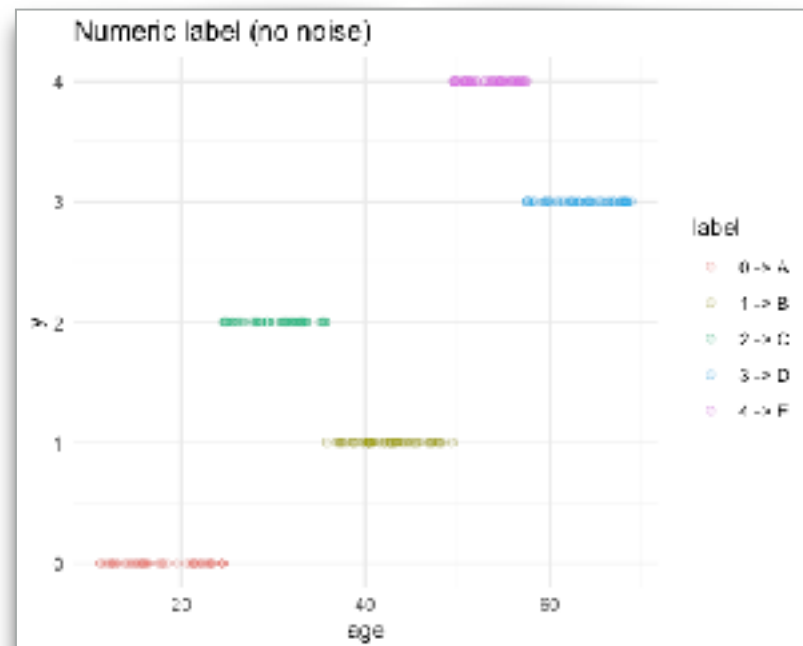
EMMA BEAUXIS-AUSSALET

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PREDICT CATEGORICAL DATA

Polynomial regressions may be fitted on categorical data, but with many parameters... but they are not interpretable.

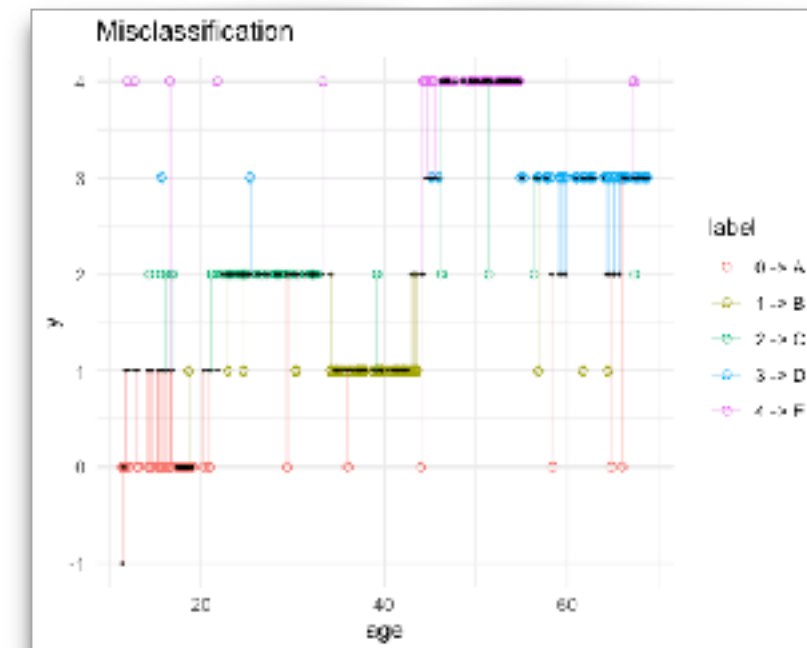
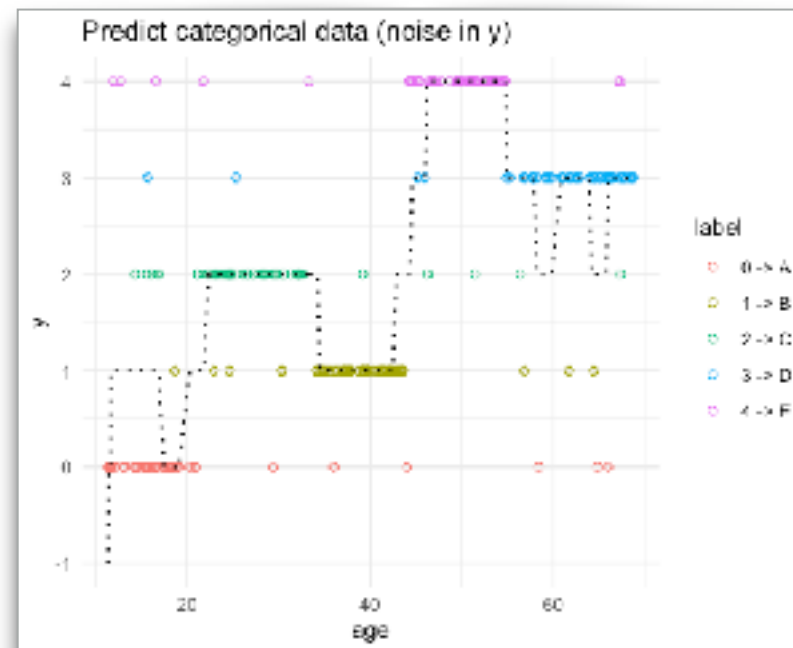
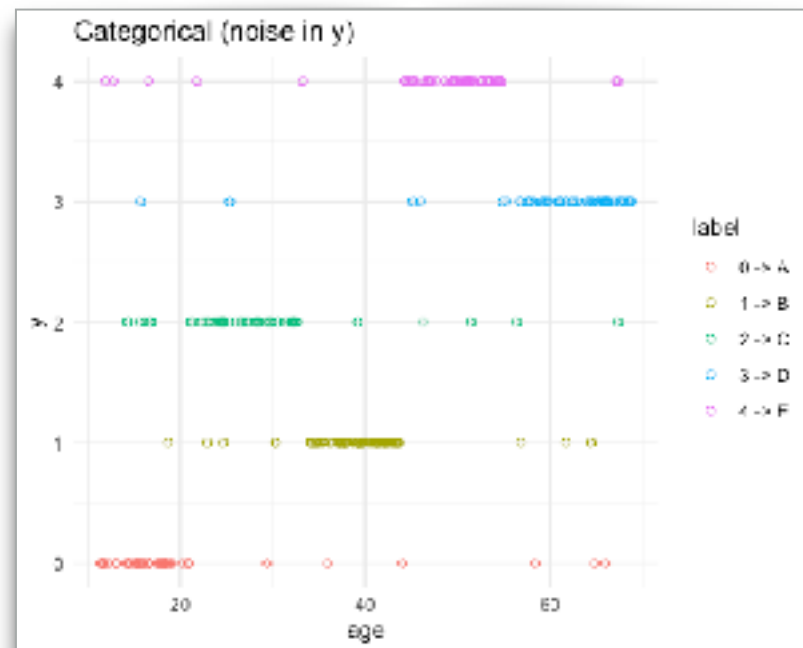
```
##### Polynomial regression - Degree 20  
model <- lm(y ~ poly(age,20), data=data)
```



PREDICT CATEGORICAL DATA

Polynomial regressions may be fitted on categorical data, but with many parameters... but they are not interpretable... and **not robust to noise!**

```
##### Polynomial regression - Degree 20  
model <- lm(y ~ poly(age,20), data=data)
```

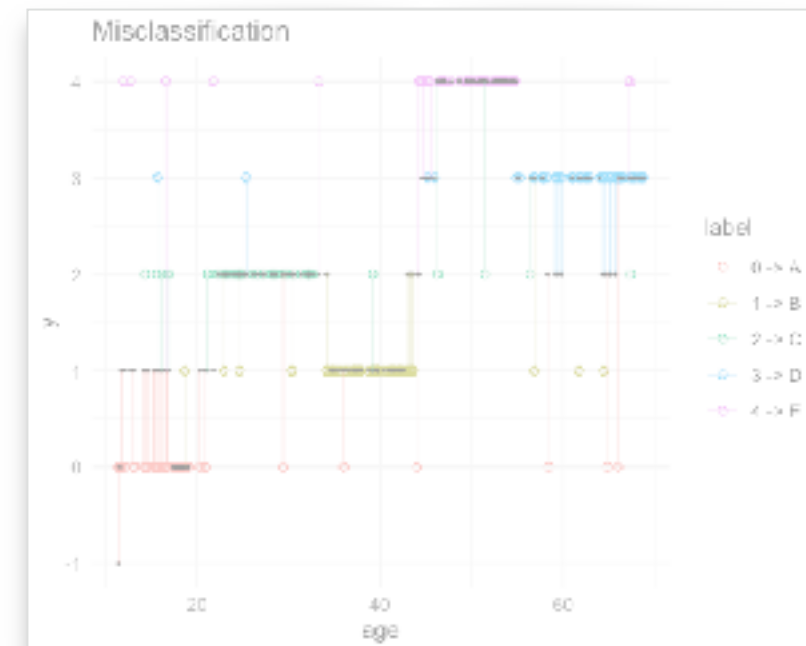
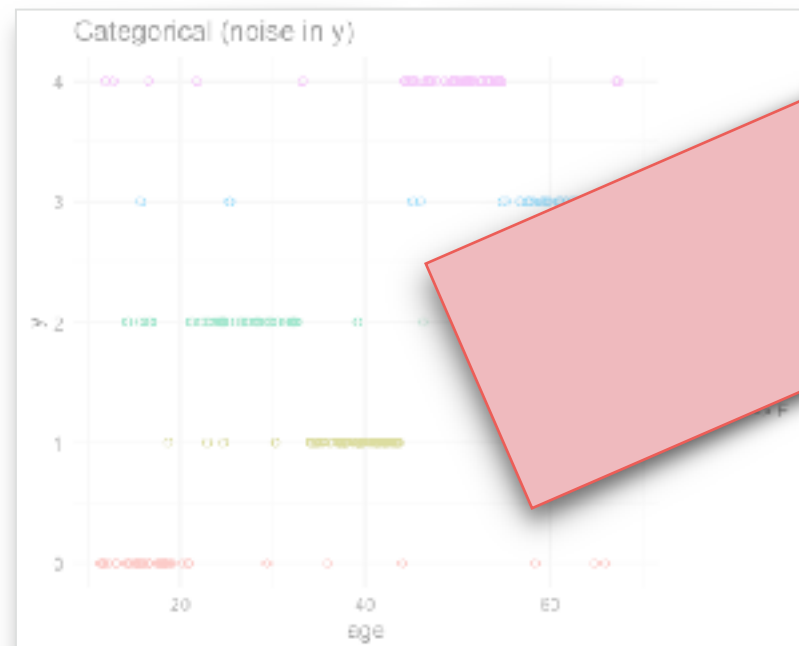


PREDICT CATEGORICAL DATA

Polynomial regressions may be fitted on categorical data but they have many parameters... but they are not interpretable... and **not robust to noise**!

```
##### Polynomial regression  
model <- lm(y ~ x, data = data)
```

We need something different !



DATA-DRIVEN TRANSFORMATION

LOGISTIC REGRESSION

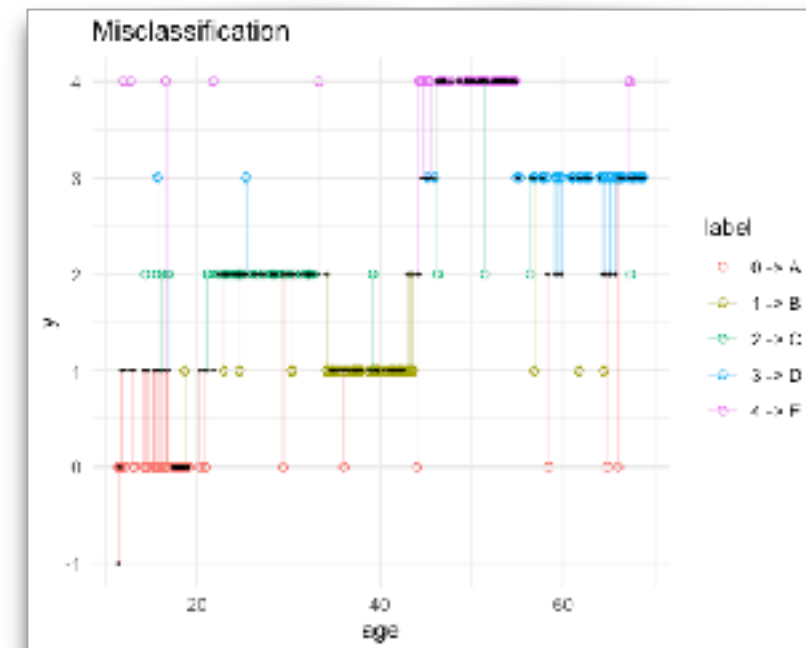
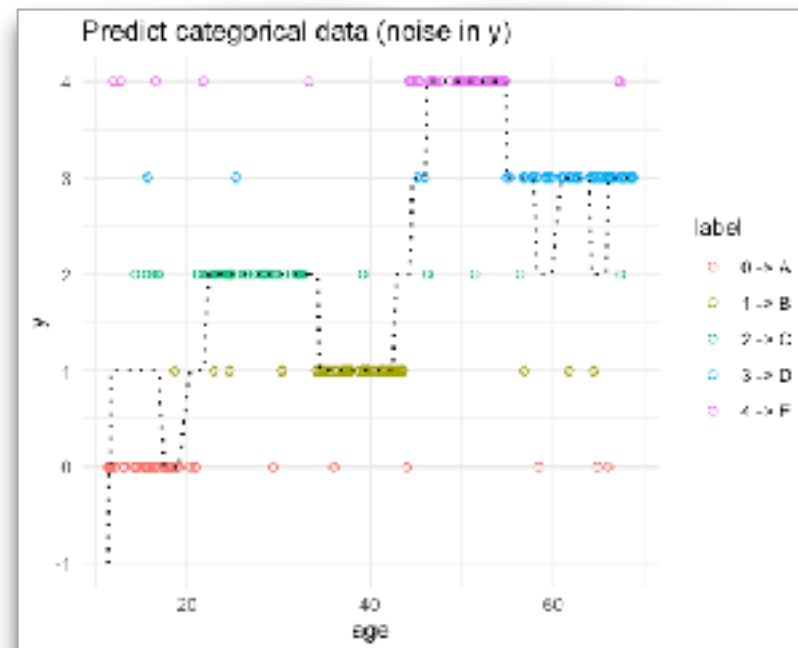
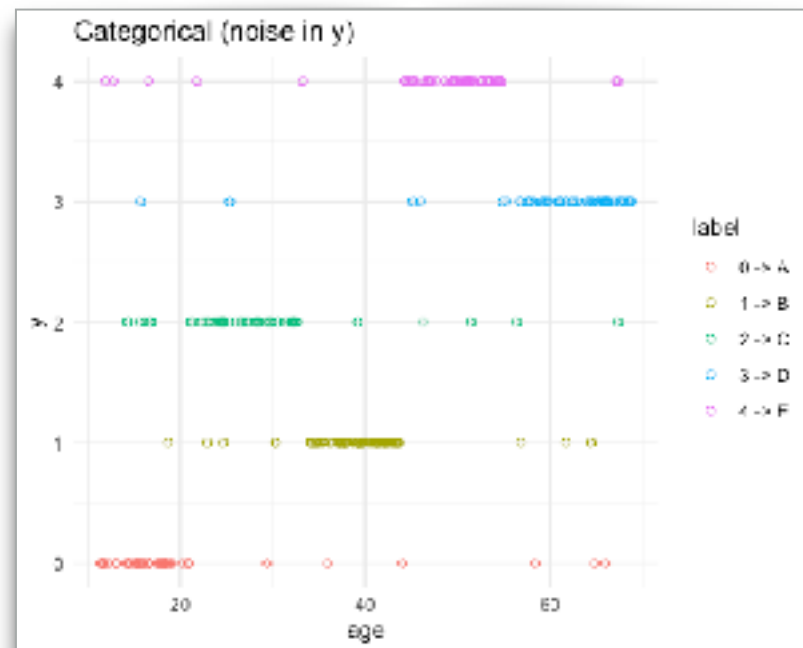
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REGRESSION ISSUES

With multiclass data, linear or polynomial regressions have **large bias**.

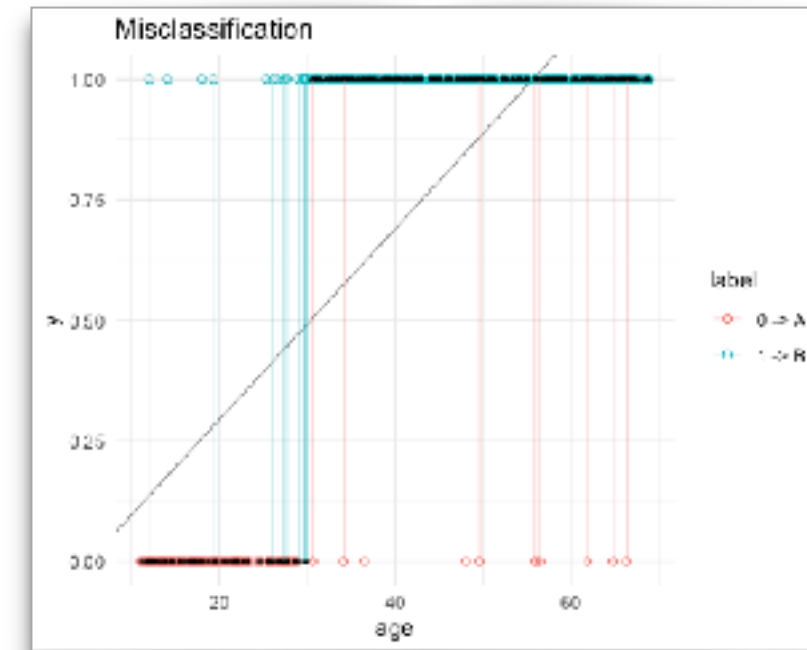
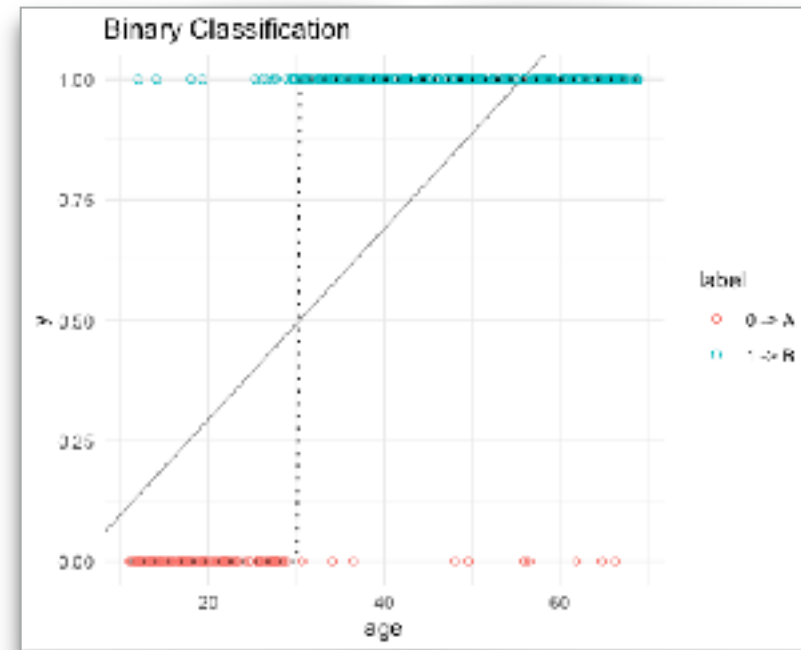
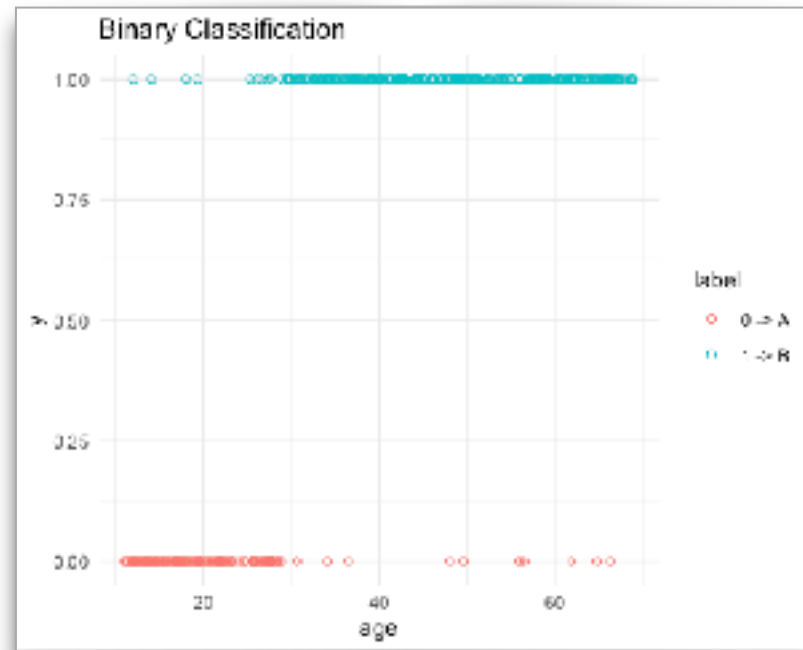
Such bias depend on how numerical class labels are ordered (arbitrarily), and on random noise.



REGRESSION ISSUES

Predictions from **linear or polynomial regressions are unbounded**, thus hard to interpret.

For instance, such models can **predict a class that does not exist**.

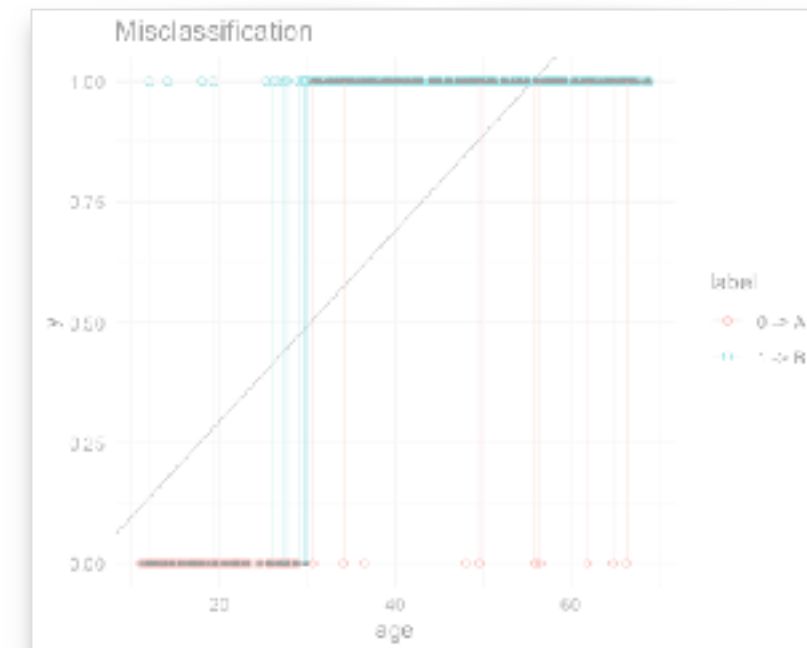
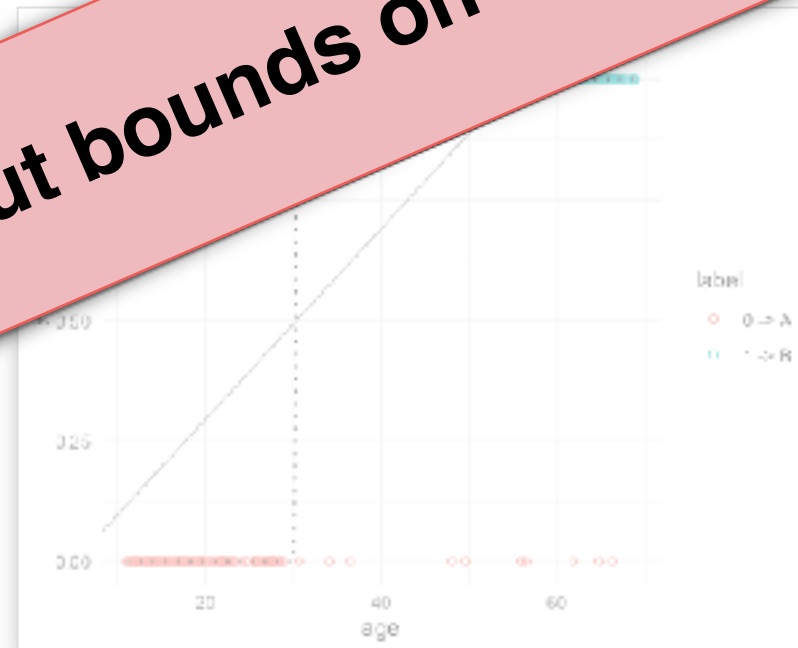
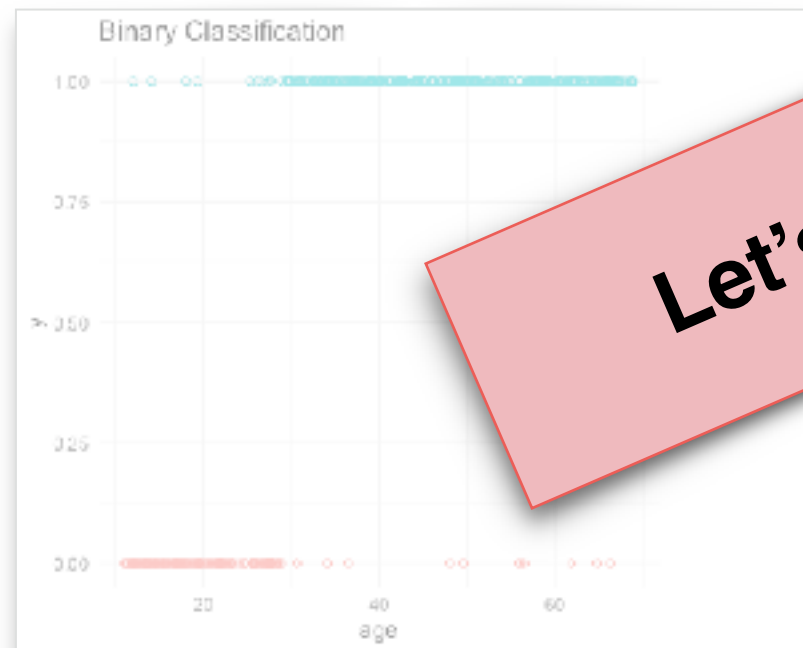


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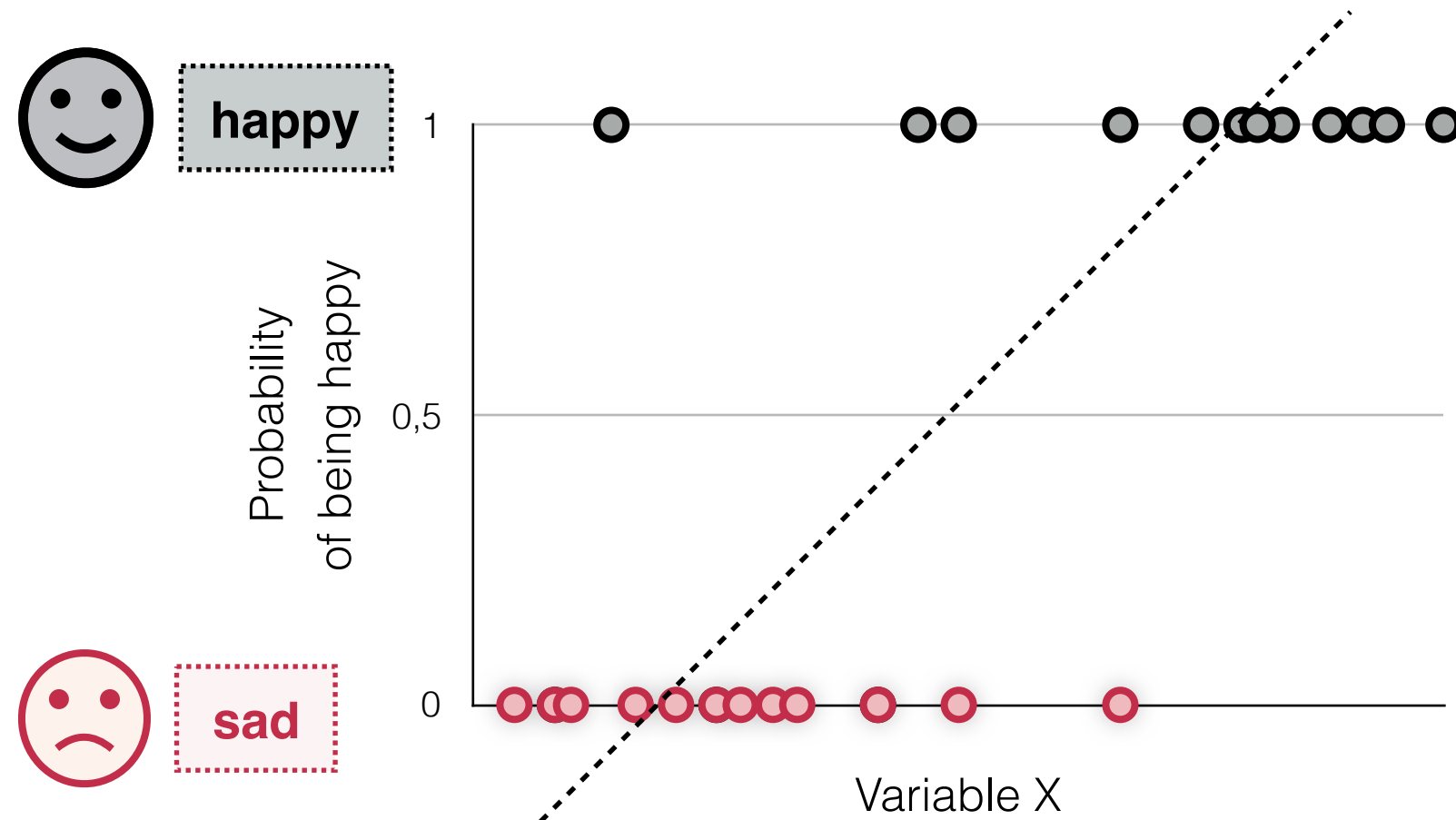
For instance, such models can **predict a class** that doesn't exist.

Let's put bounds on the predictions....



LOGISTIC REGRESSION

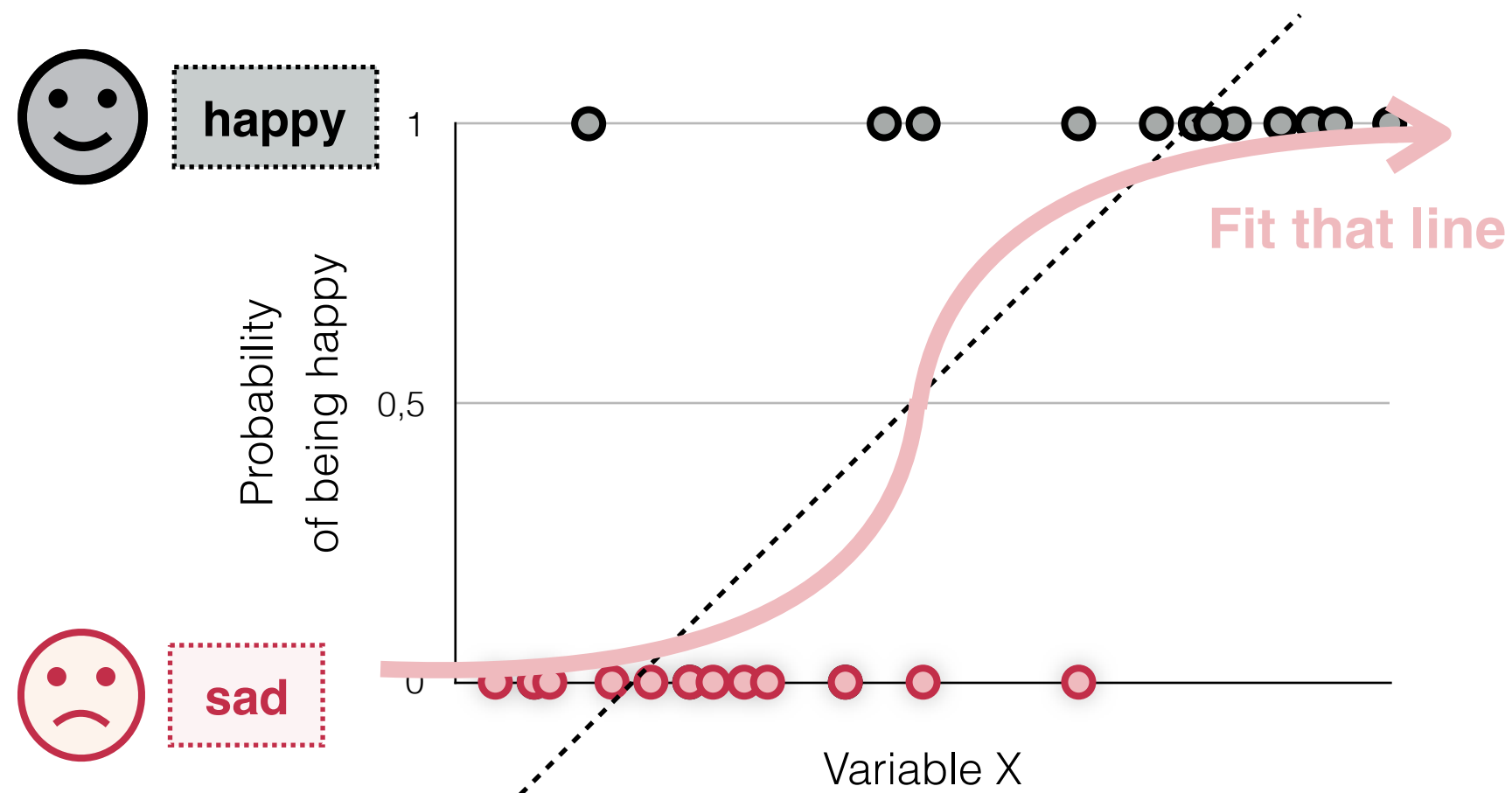
We can transform linear regressions into a **sigmoid function**, a s-shaped line, using the slope and intercept of the straight line.



$$\hat{y} = a\mathbf{x} + b$$

LOGISTIC REGRESSION

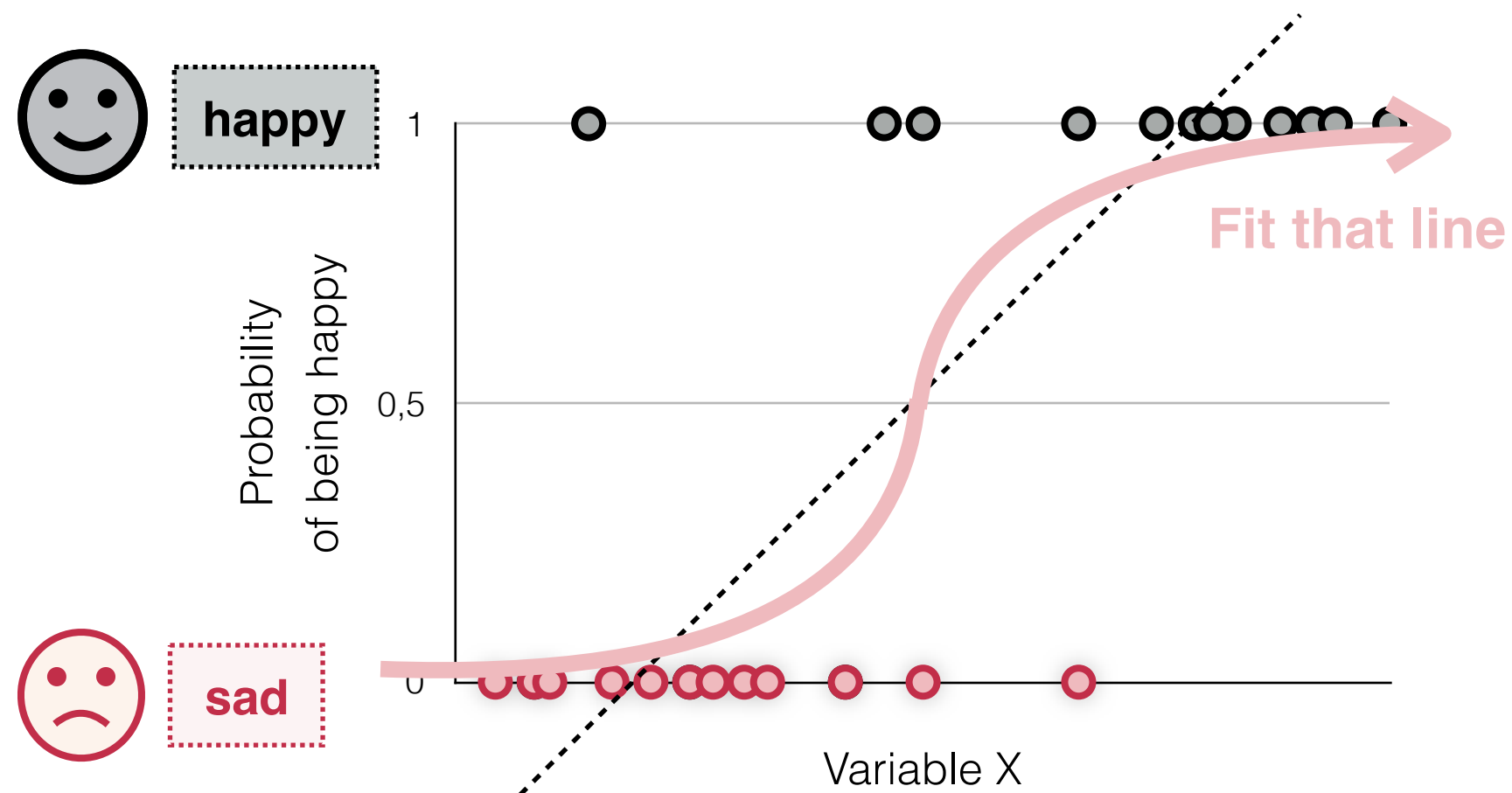
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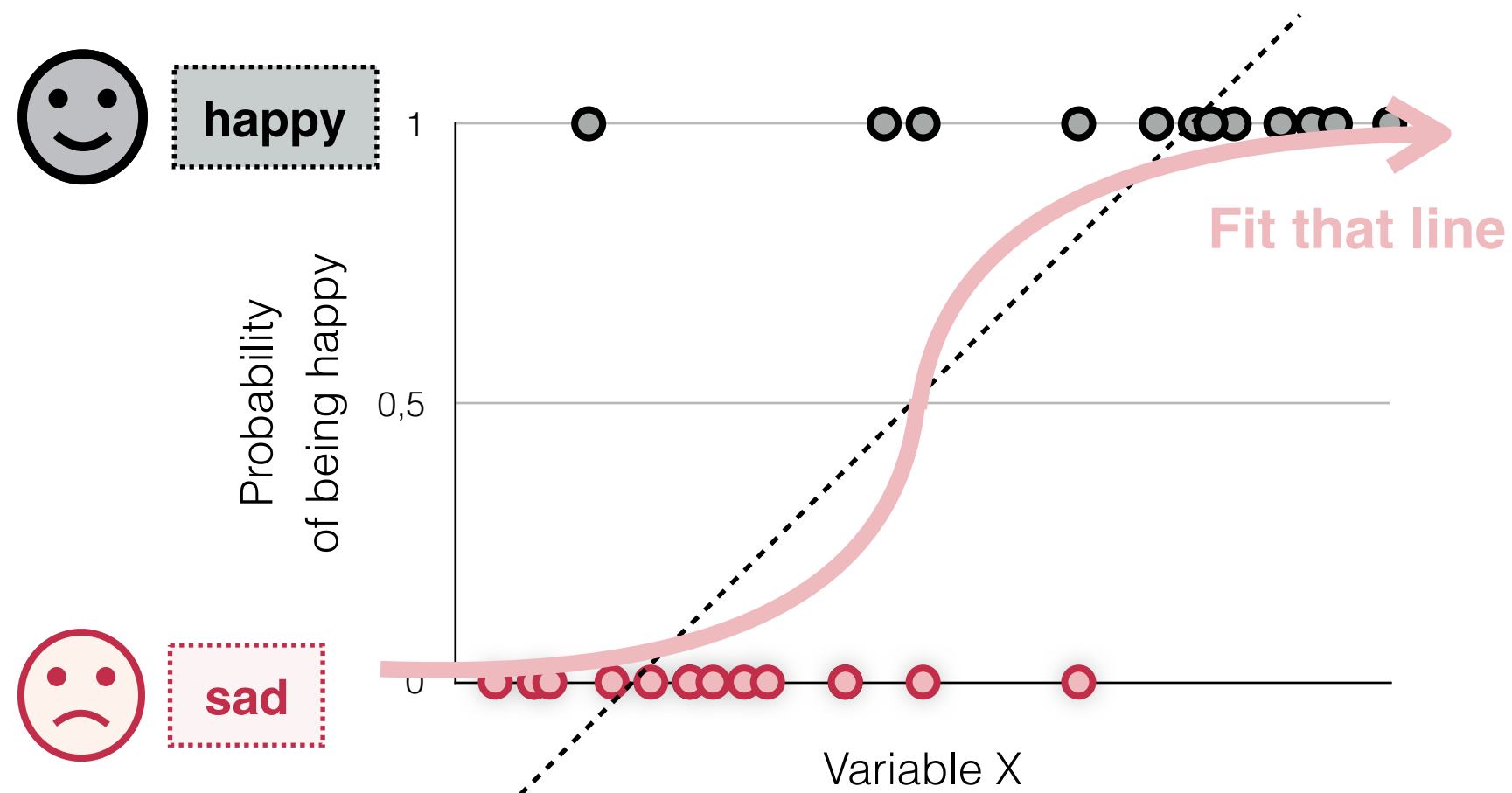


$$\hat{y} = a\mathbf{x} + b$$

$$\hat{P}(\text{happy}) = \frac{1}{1 + e^{-(a\mathbf{x}+b)}}$$

LOGISTIC REGRESSION

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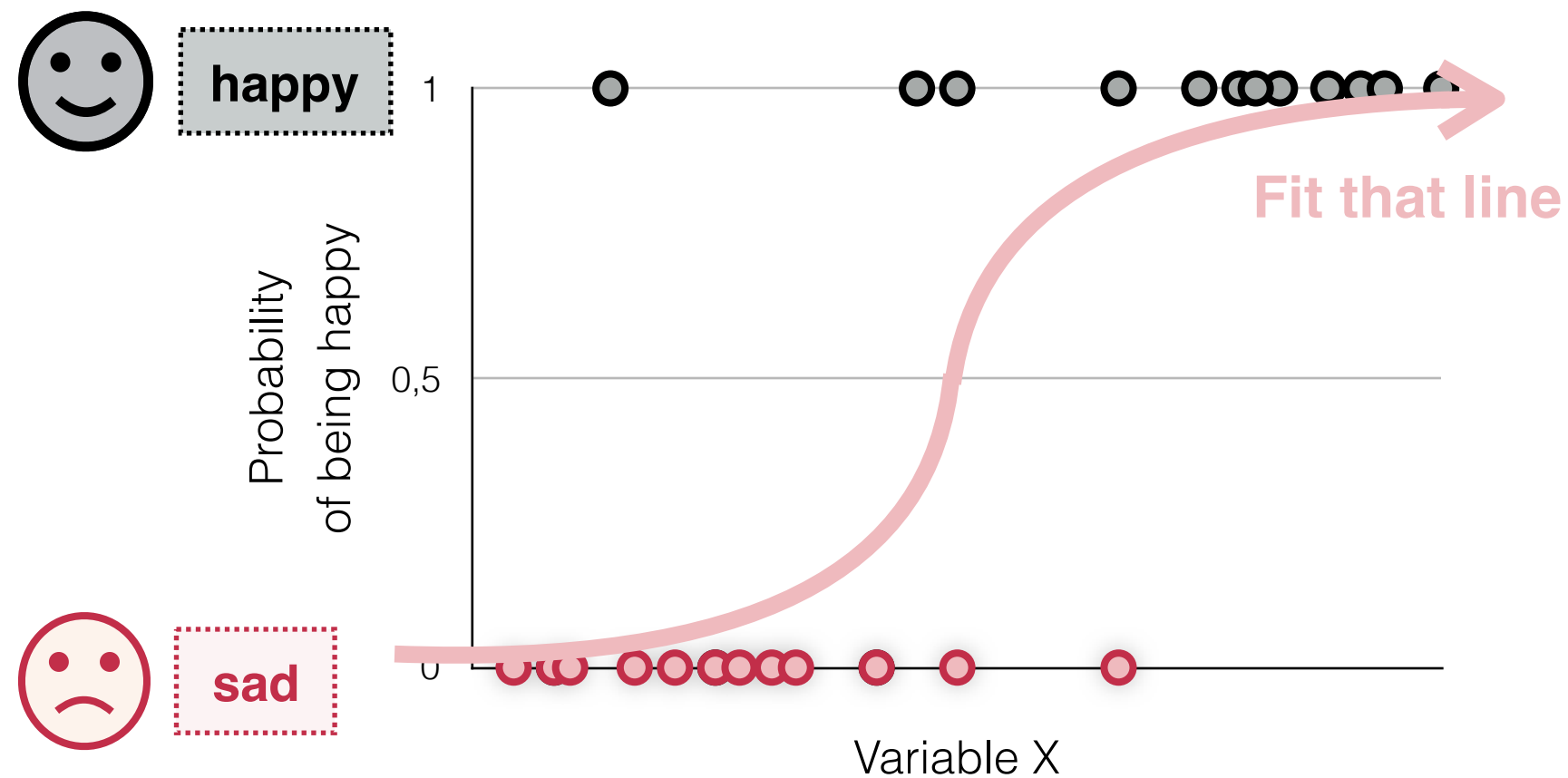


$$\hat{y} = a\mathbf{x} + b$$

$$\hat{P}(\text{happy}) = \frac{1}{1 + e^{-(a\mathbf{x} + b)}}$$

LOGISTIC REGRESSION

The sigmoid function has **bonded results** that can be **interpreted as probabilities** of class membership (the probability that a data point belongs to a class).



$$\hat{P}(happy) = \frac{1}{1 + e^{-(ax+b)}}$$

LOGISTIC REGRESSION

$$\hat{P}(\textit{neutral}) = \frac{1}{1 + e^{-(\alpha_0 + \alpha_1 \mathbf{x})}}$$

$$\hat{P}(\textit{happy}) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 \mathbf{x})}}$$

$$\hat{P}(\textit{sad}) = \frac{1}{1 + e^{-(\zeta_0 + \zeta_1 \mathbf{x})}}$$

Multiclass problems require
one regression model per class.

The predicted class is the one having
the maximum probability.

LOGISTIC REGRESSION

$$\hat{P}(\text{neutral}) = \frac{1}{1 + e^{-(\alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \dots)}}$$

$$\hat{P}(\text{happy}) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots)}}$$

$$\hat{P}(\text{sad}) = \frac{1}{1 + e^{-(\zeta_0 + \zeta_1 x_1 + \zeta_2 x_2 + \dots)}}$$

Multivariate logistic regressions add variables **within the embedded linear regression**.

DATA-DRIVEN TRANSFORMATION

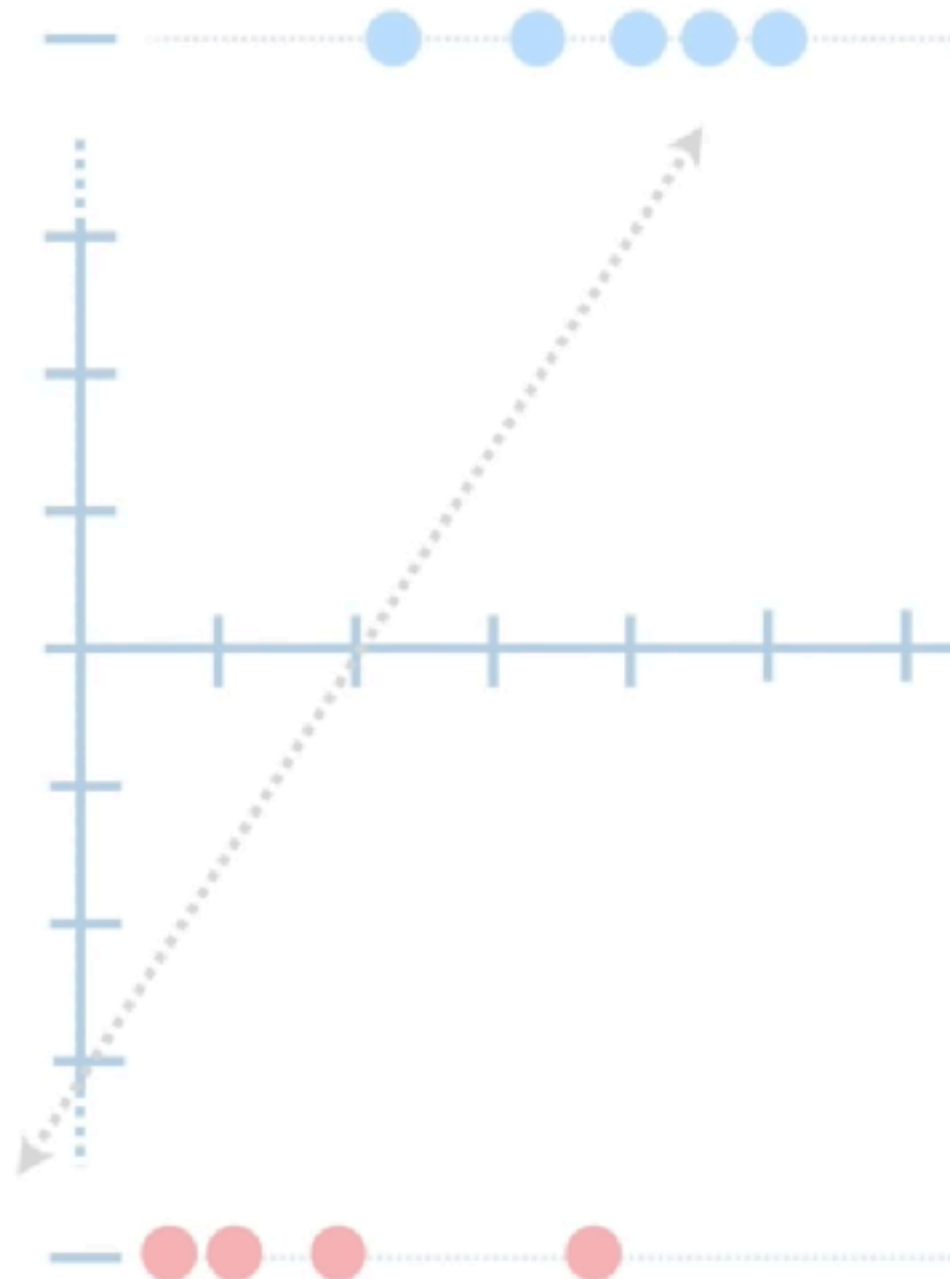
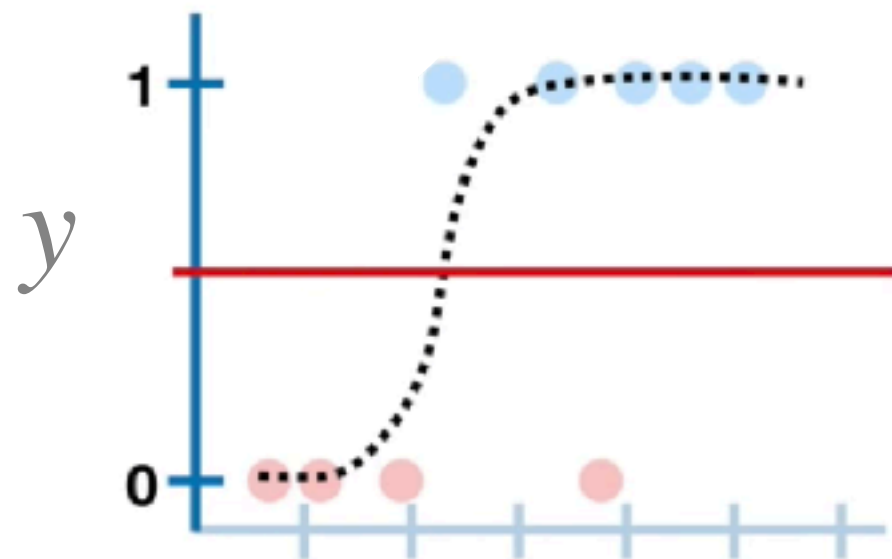
DEEP DIVE IN LOGISTIC REGRESSION

EMMA BEAUXIS-AUSSALET

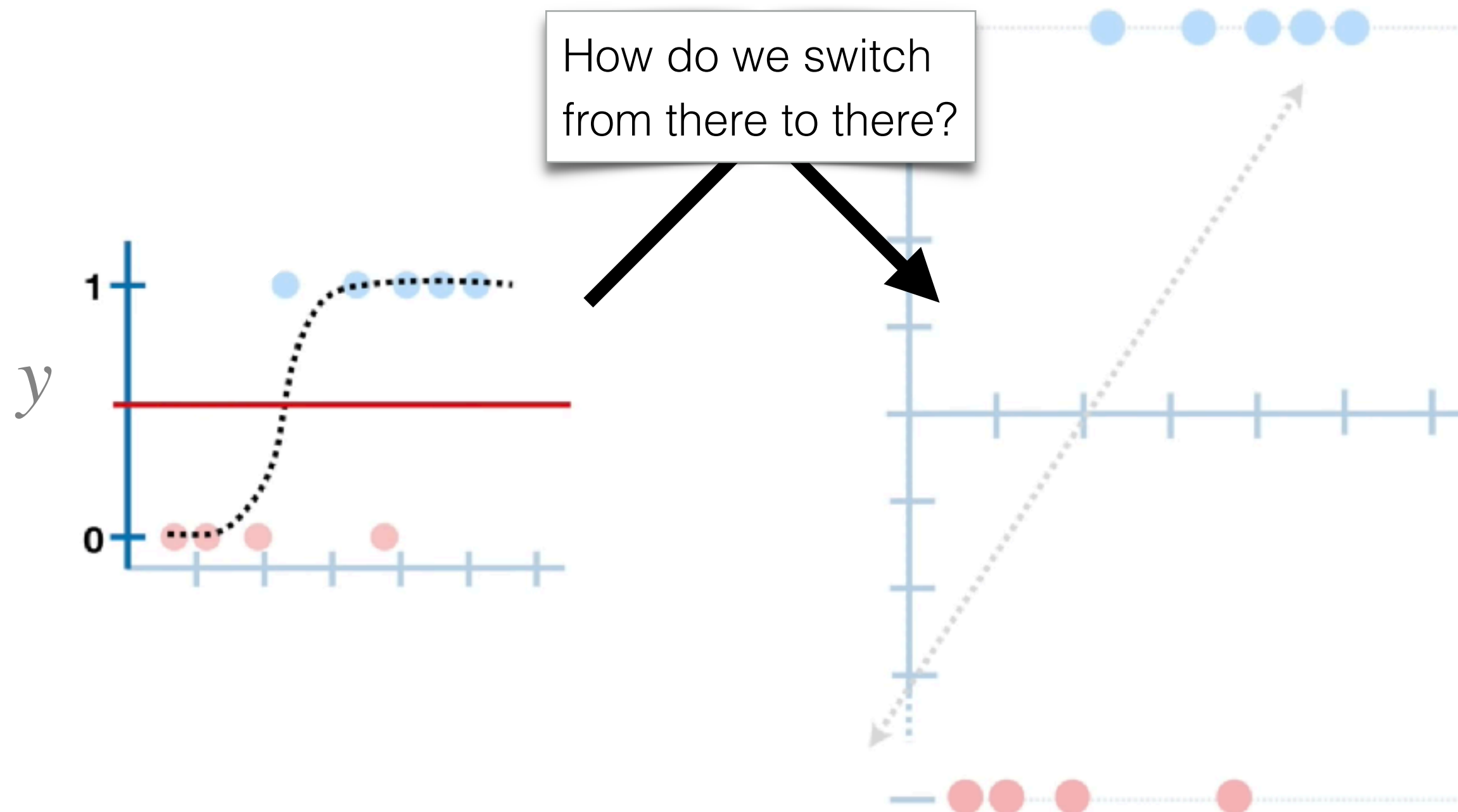
e.m.a.l.beauxis@hva.nl

HOW DOES IT WORK ?

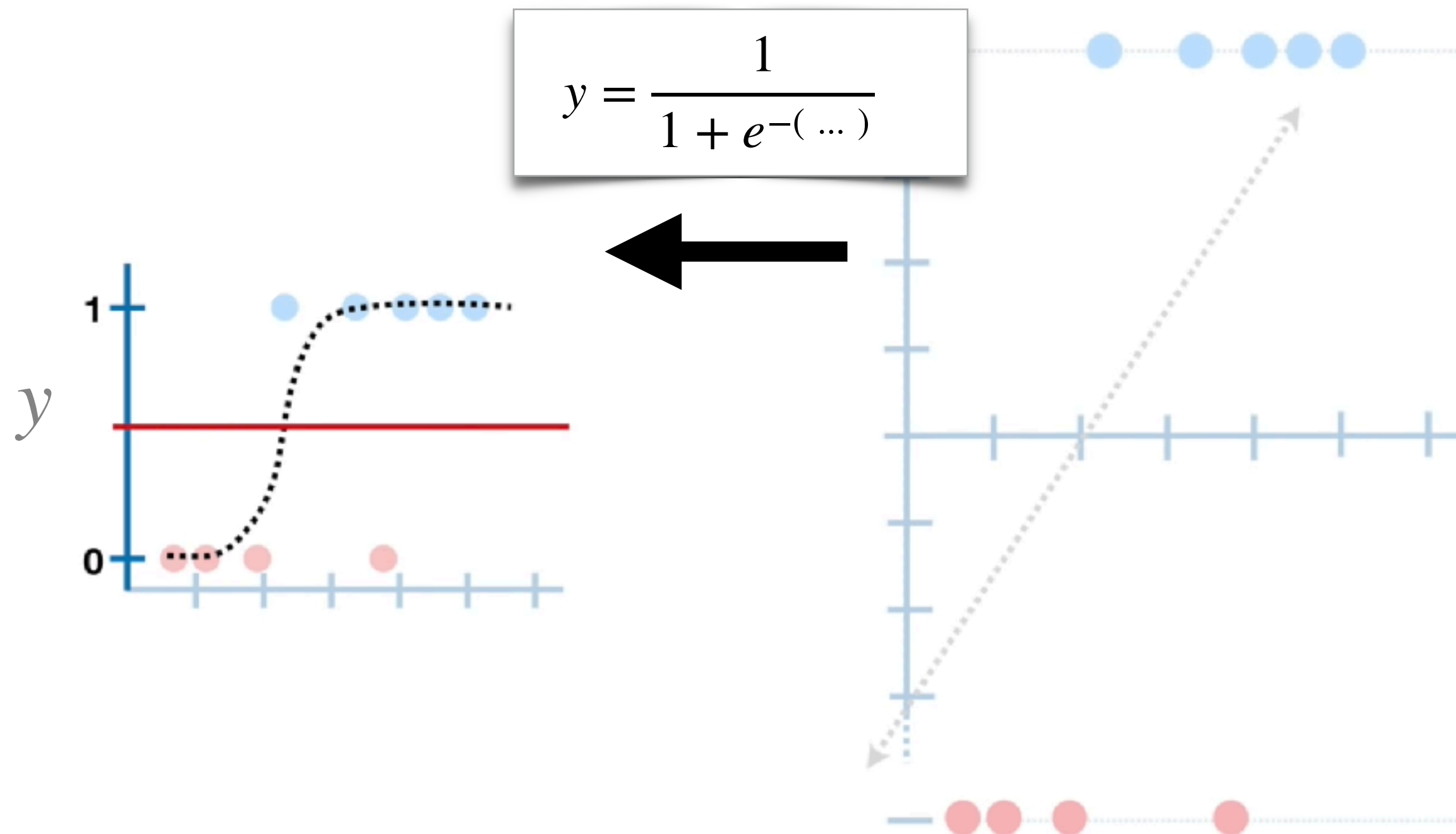
It is just fitting a **straight regression line**,
...but after transforming the data.



HOW DOES IT WORK ?

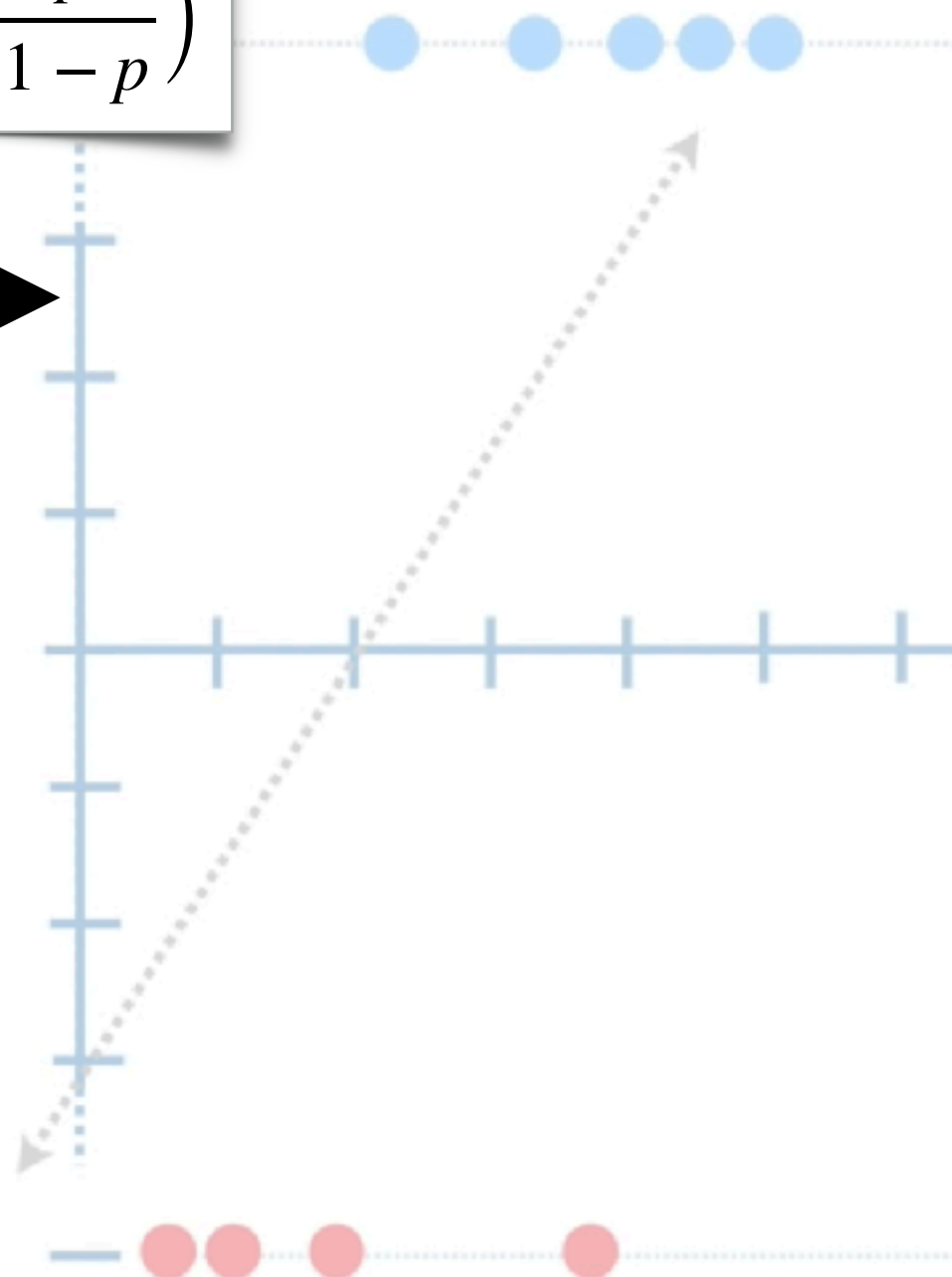
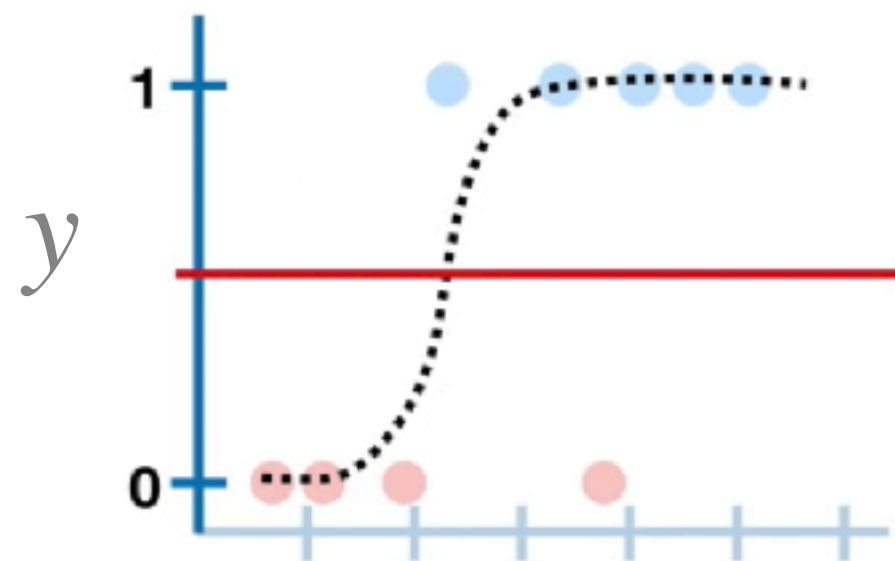


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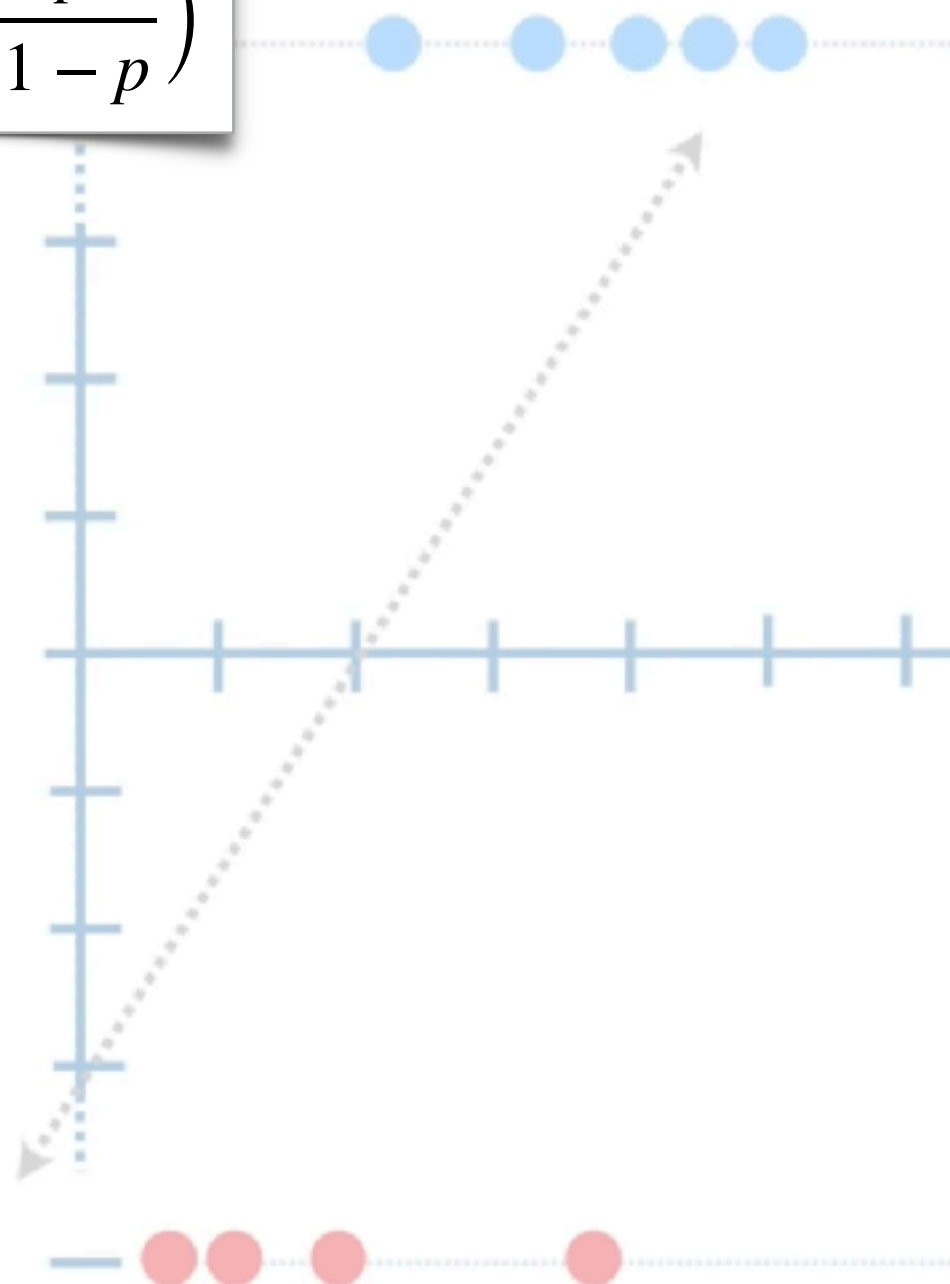
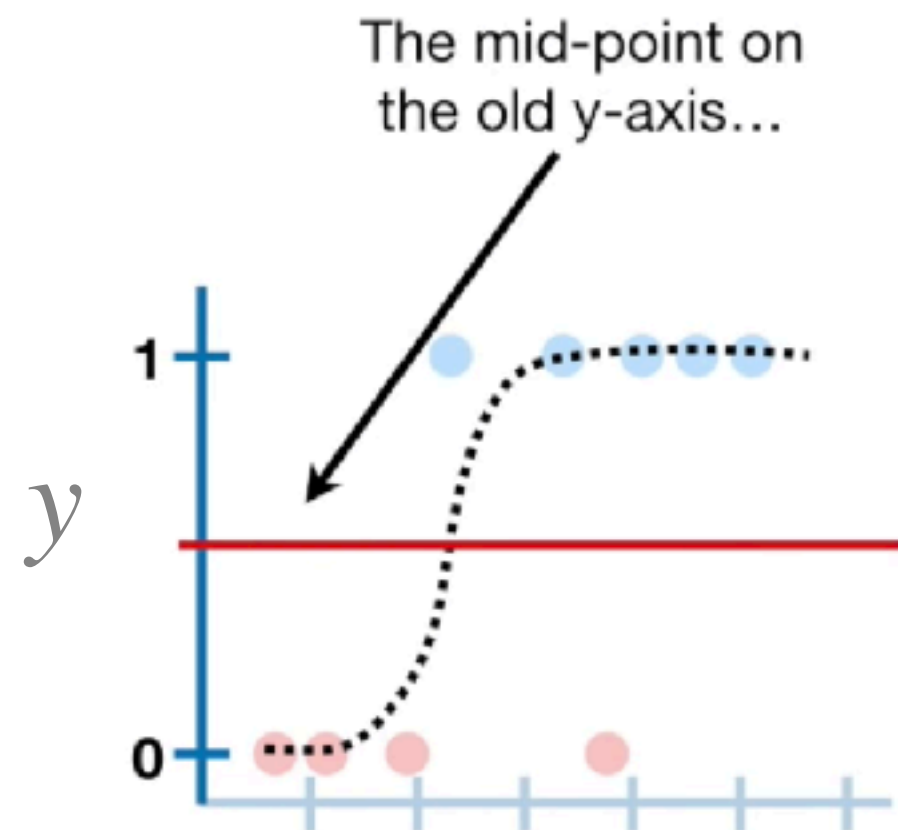
HOW DOES IT WORK ?

$$y_{trans} = \log\left(\frac{p}{1-p}\right)$$



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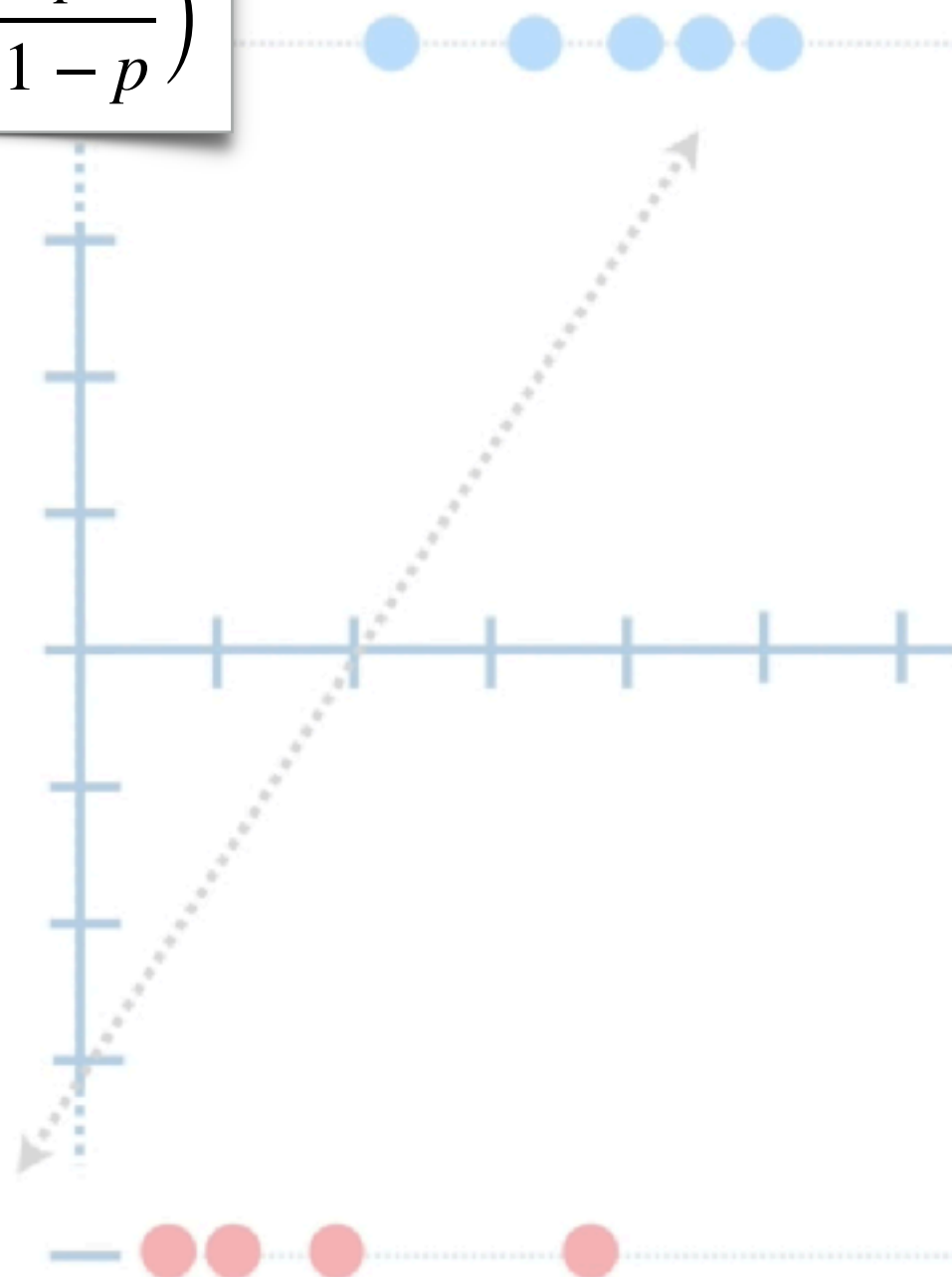
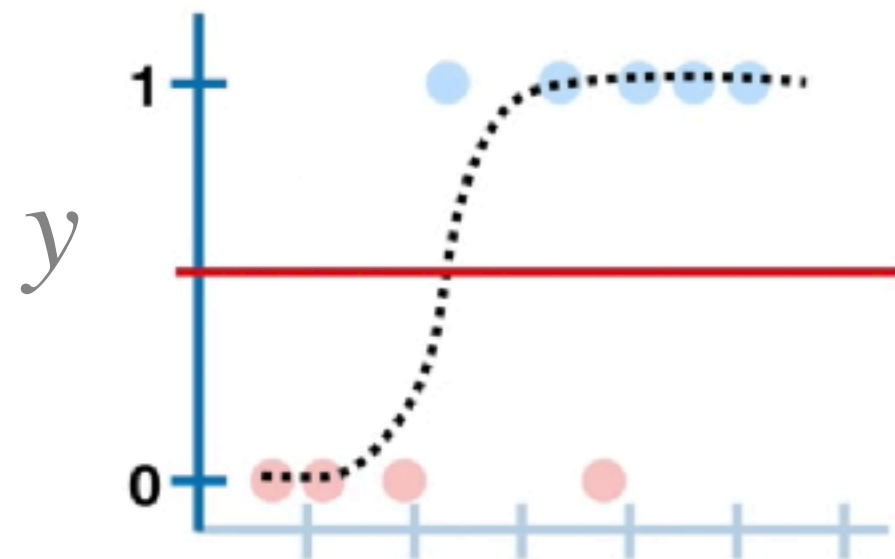


HOW DOES IT WORK ?

$$y_{trans} = \log\left(\frac{p}{1-p}\right)$$

...and when we
plug $p = 0.5$ into
the logit formula
and do the math...

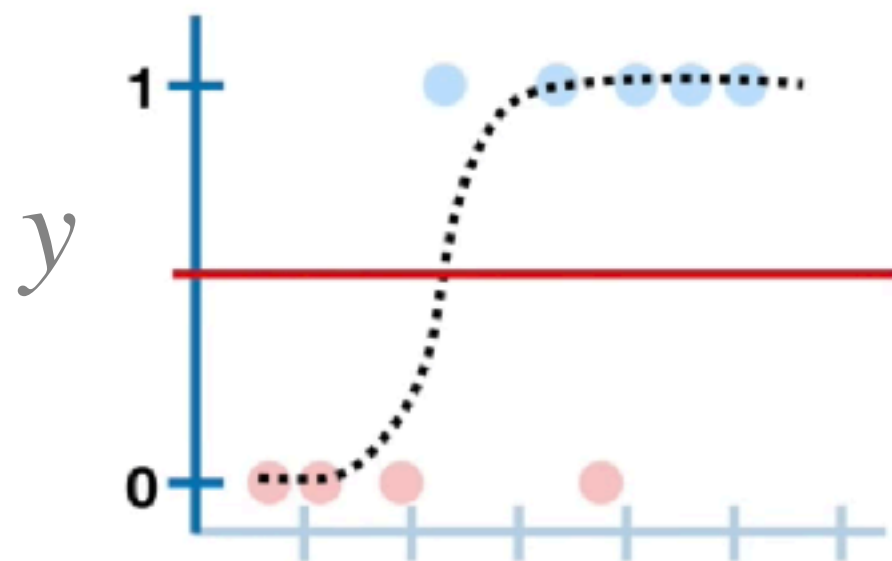
$$\log\left(\frac{0.5}{0.5}\right)$$



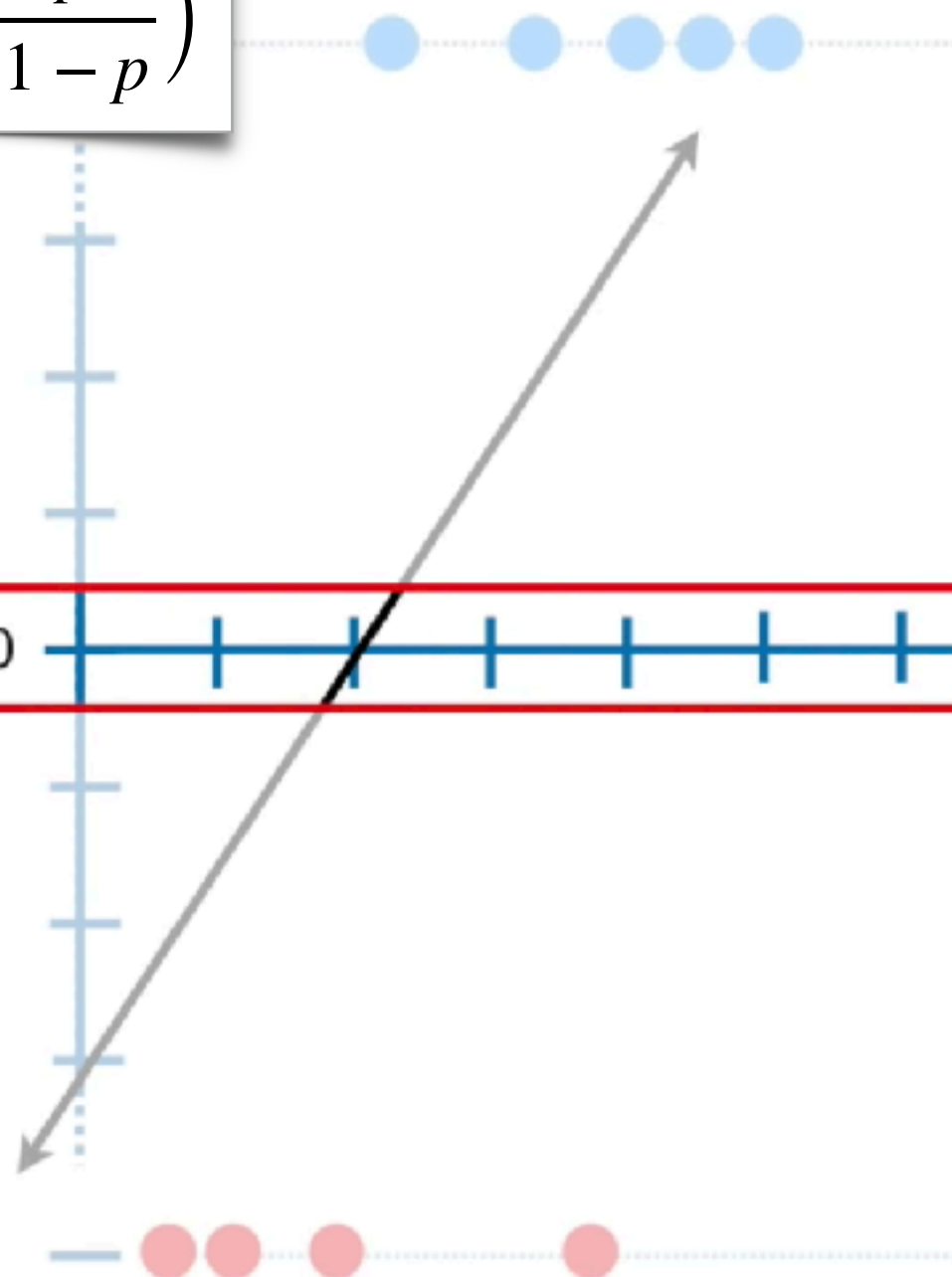
HOW DOES IT WORK ?

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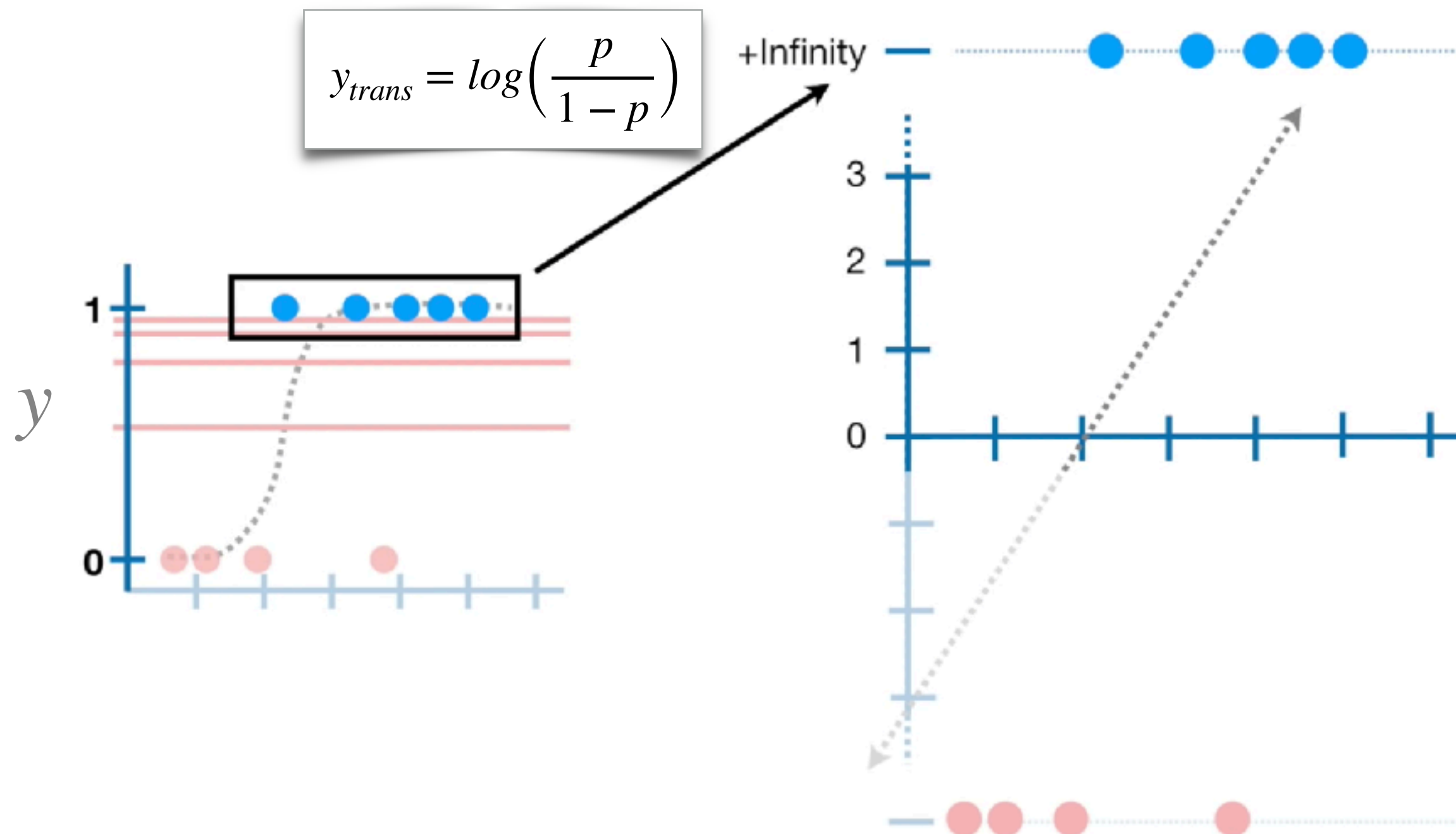
...we get 0, the
center of the new
y-axis.



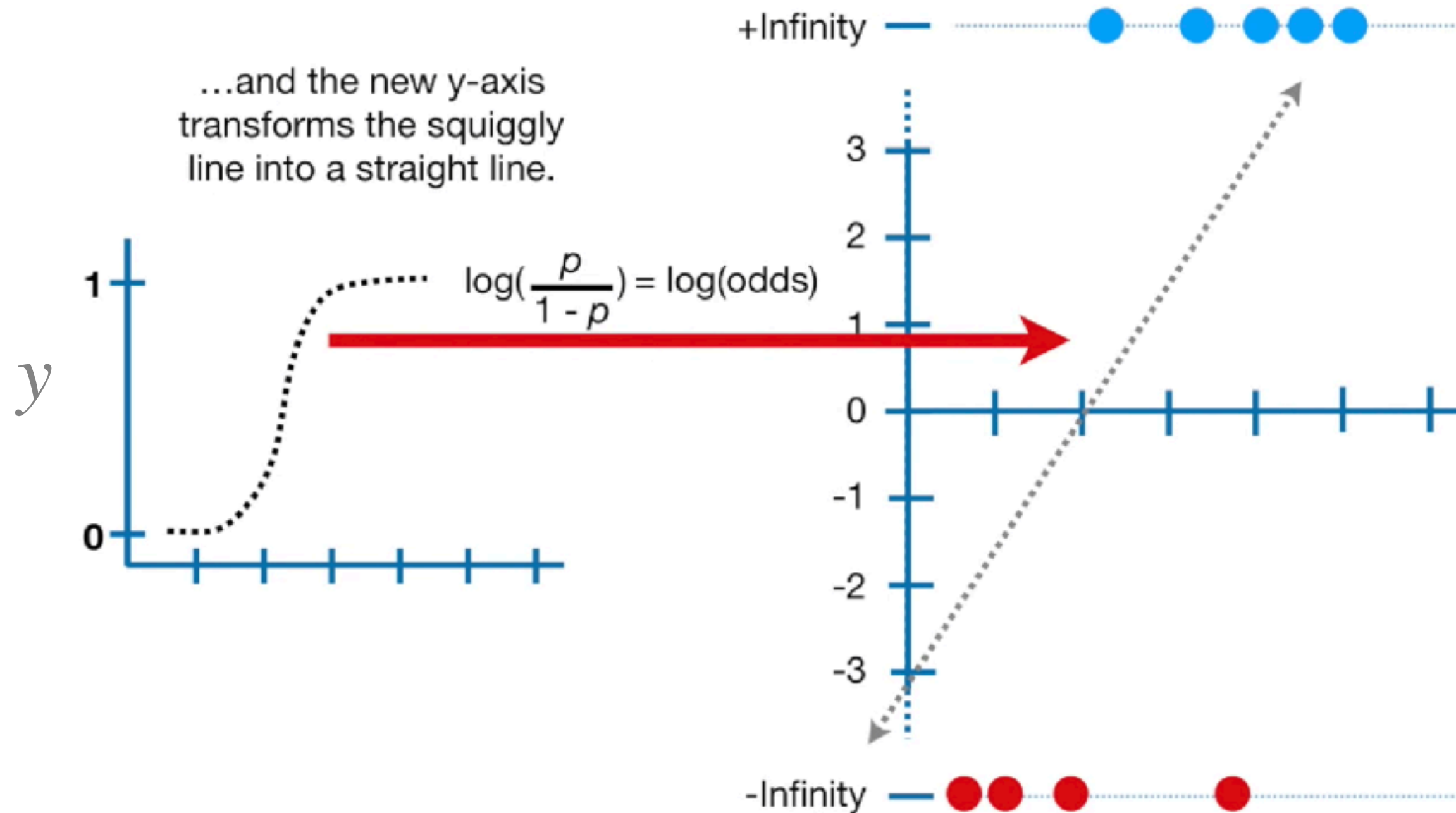
$$\log(1) = 0$$



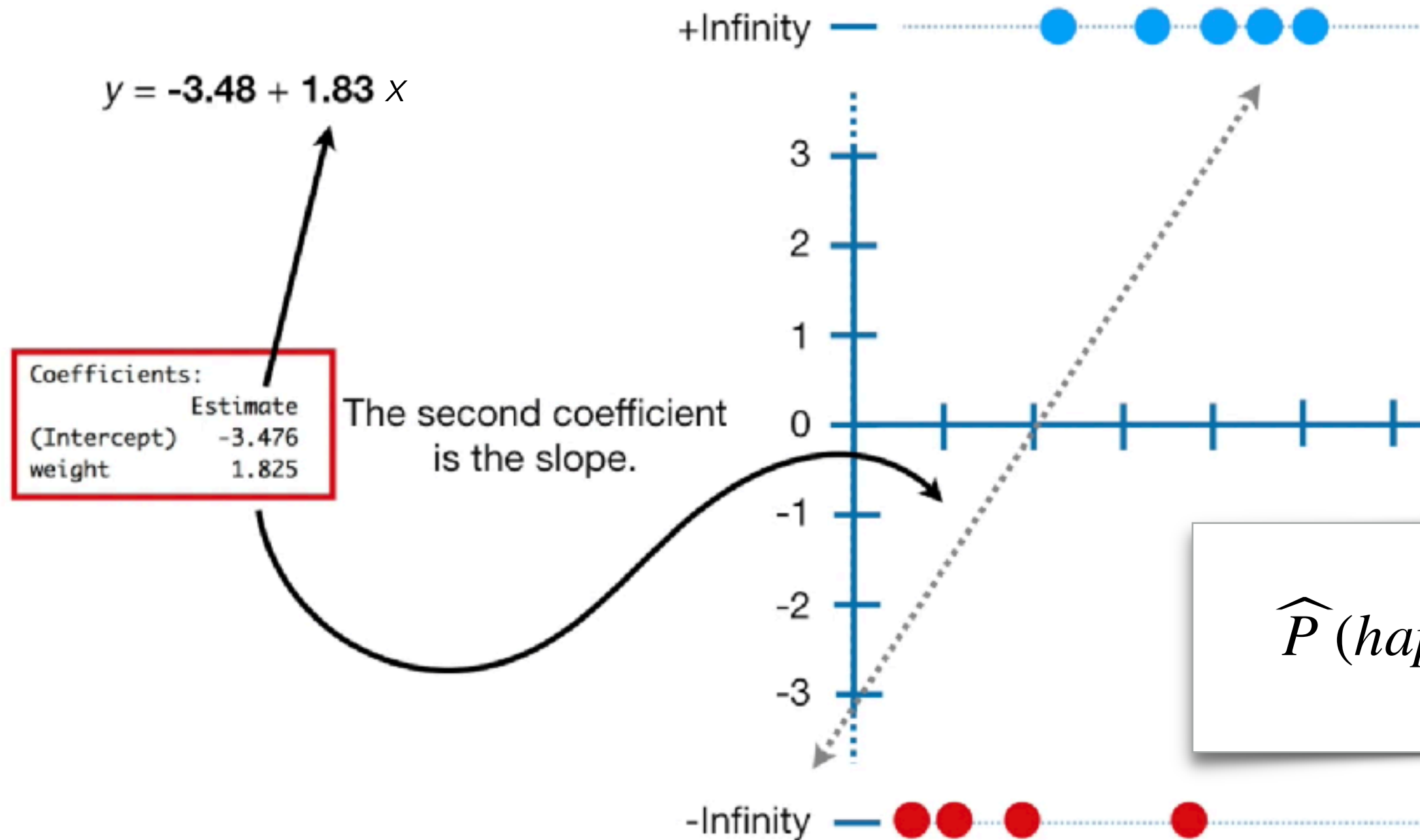
HOW DOES IT WORK ?



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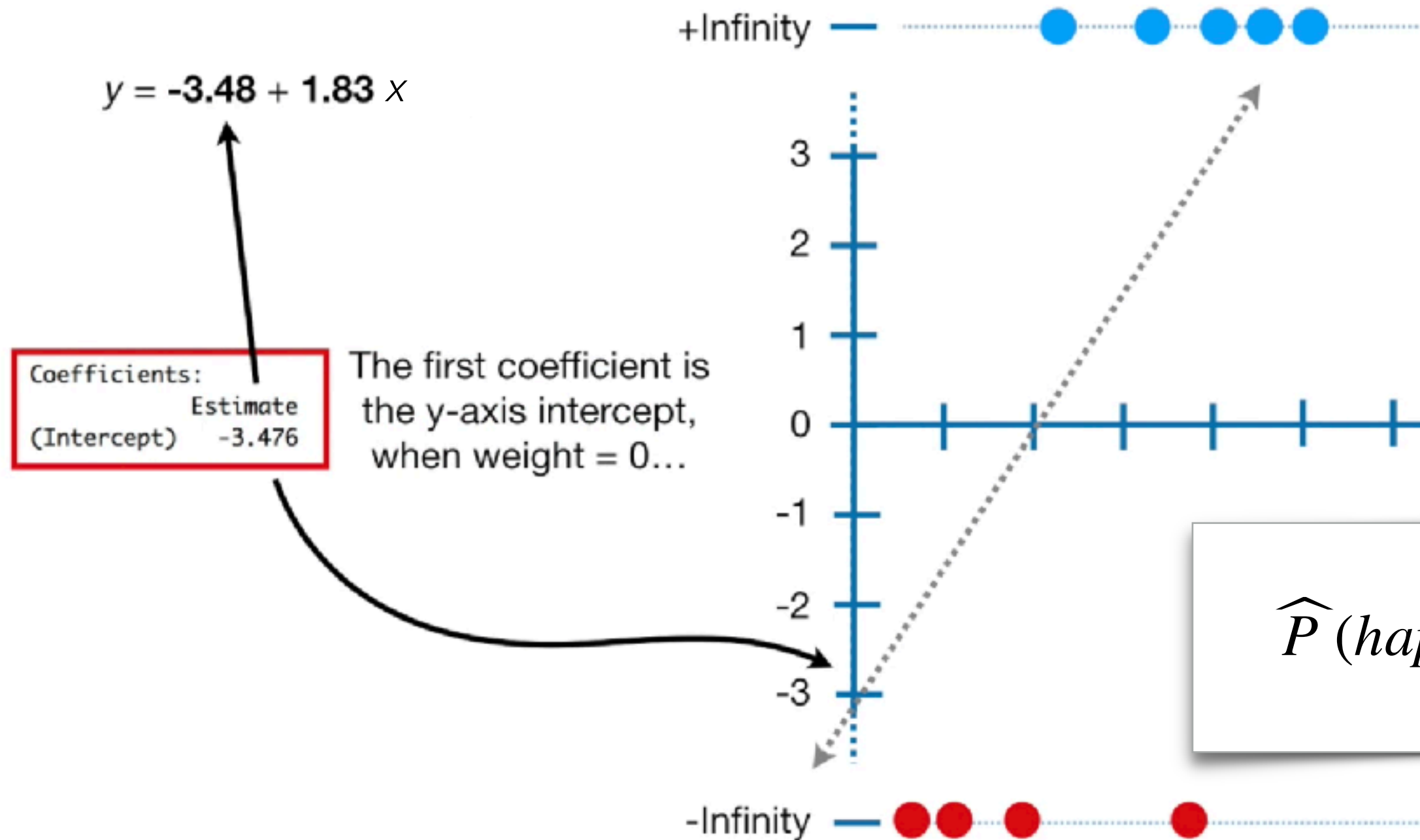


HOW DOES IT WORK ?



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DATA-DRIVEN TRANSFORMATION

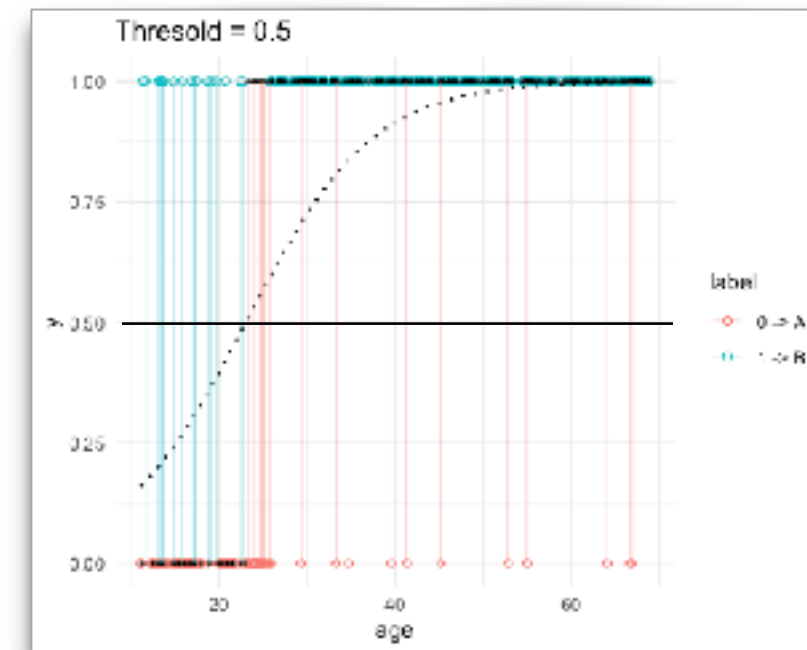
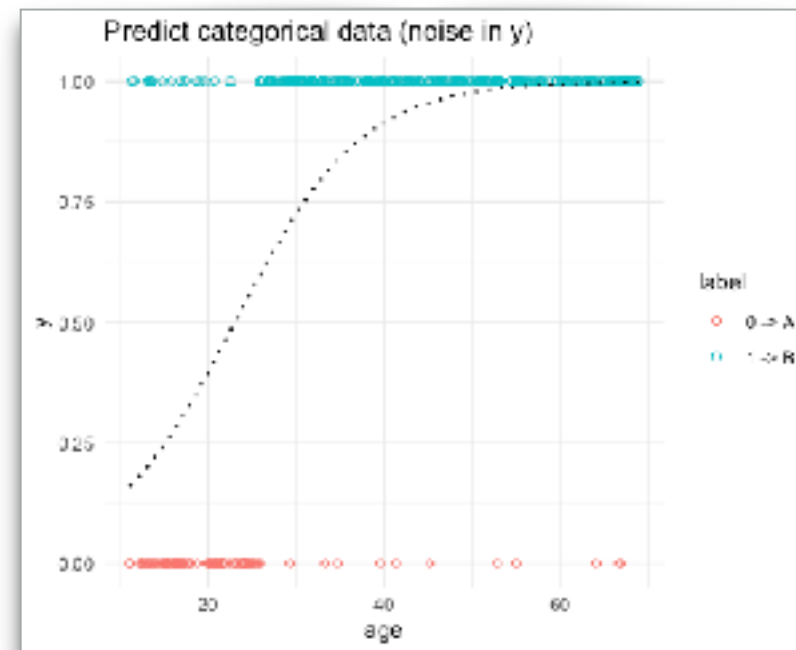
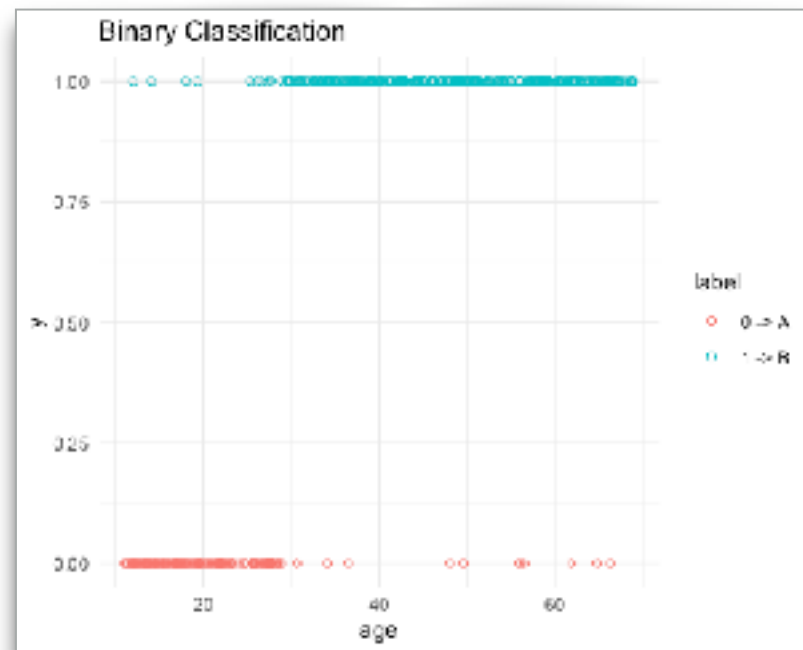
EXAMPLE

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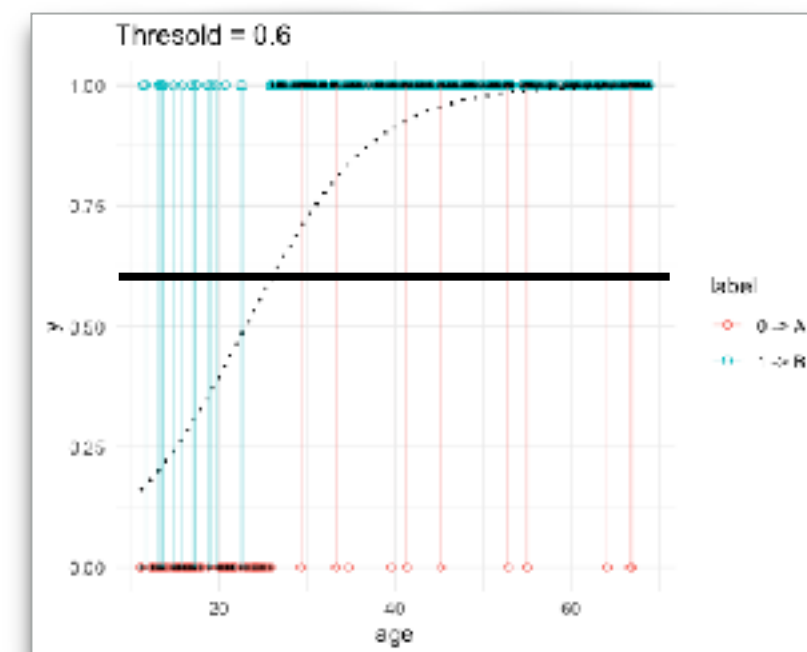
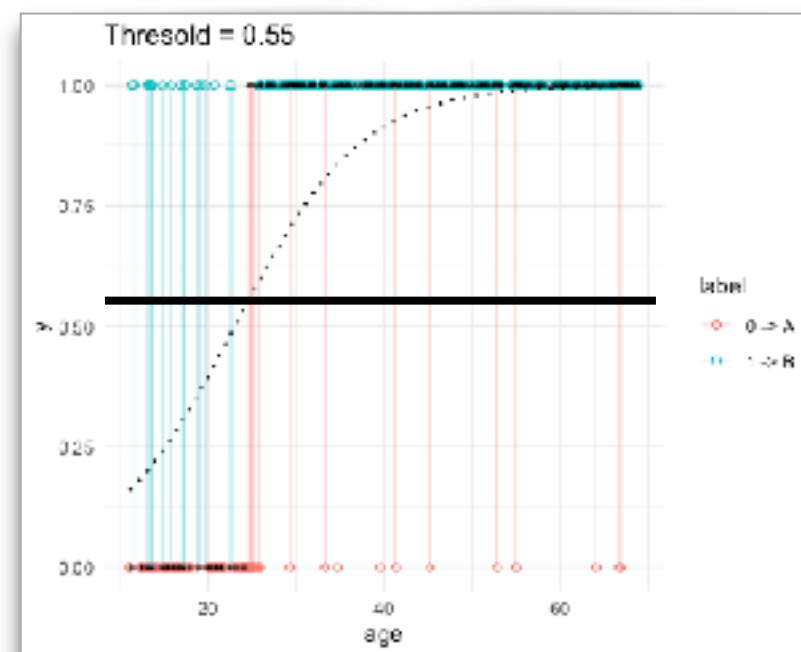
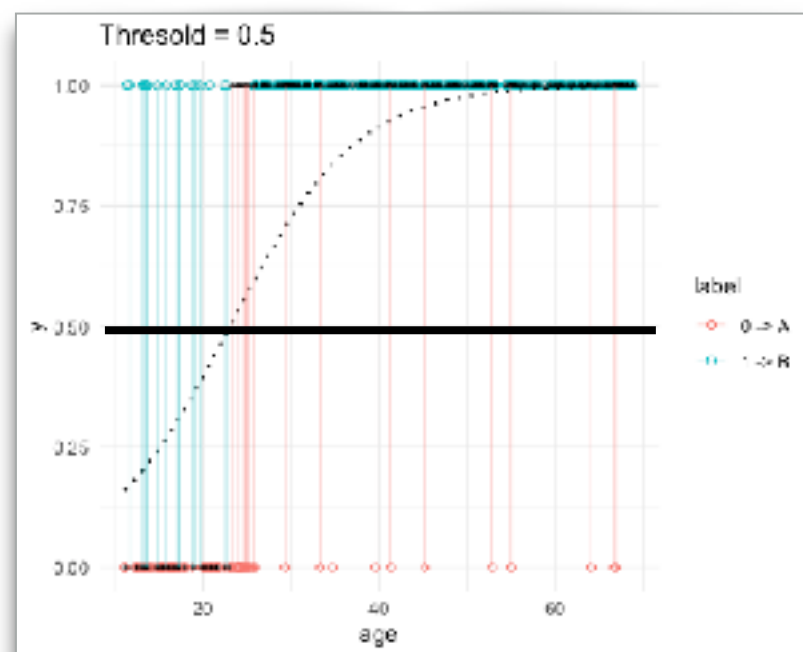
EXAMPLE WITH BINARY DATA

```
##### Logistic regression #####  
model <- glm( formula = y ~ age,  
              family = binomial,  
              data = data)
```



EXAMPLE WITH BINARY DATA

Different **thresholds can be used** to assign the final class to each data points.



DATA-DRIVEN TRANSFORMATION

QUESTION / DISCUSSION

